

12:00 - 1:00	intro: the why of normative approaches part 1: sensory coding
1:20 - 2:20	part 2: inference part 3: action selection
2:40 - 3:40	hands-on problems
3:40 - 4:00	outlook

wifi | Hilton Honors Meeting
csnventi20

TAs

Diego Arribas

Zoe Ashwood

Pierre-Etienne Fiquet

Caroline Haimerl

Anna Kutschireiter

Tzuhsuan Ma (Maz)

Jorge Menendez

Josue Nassar

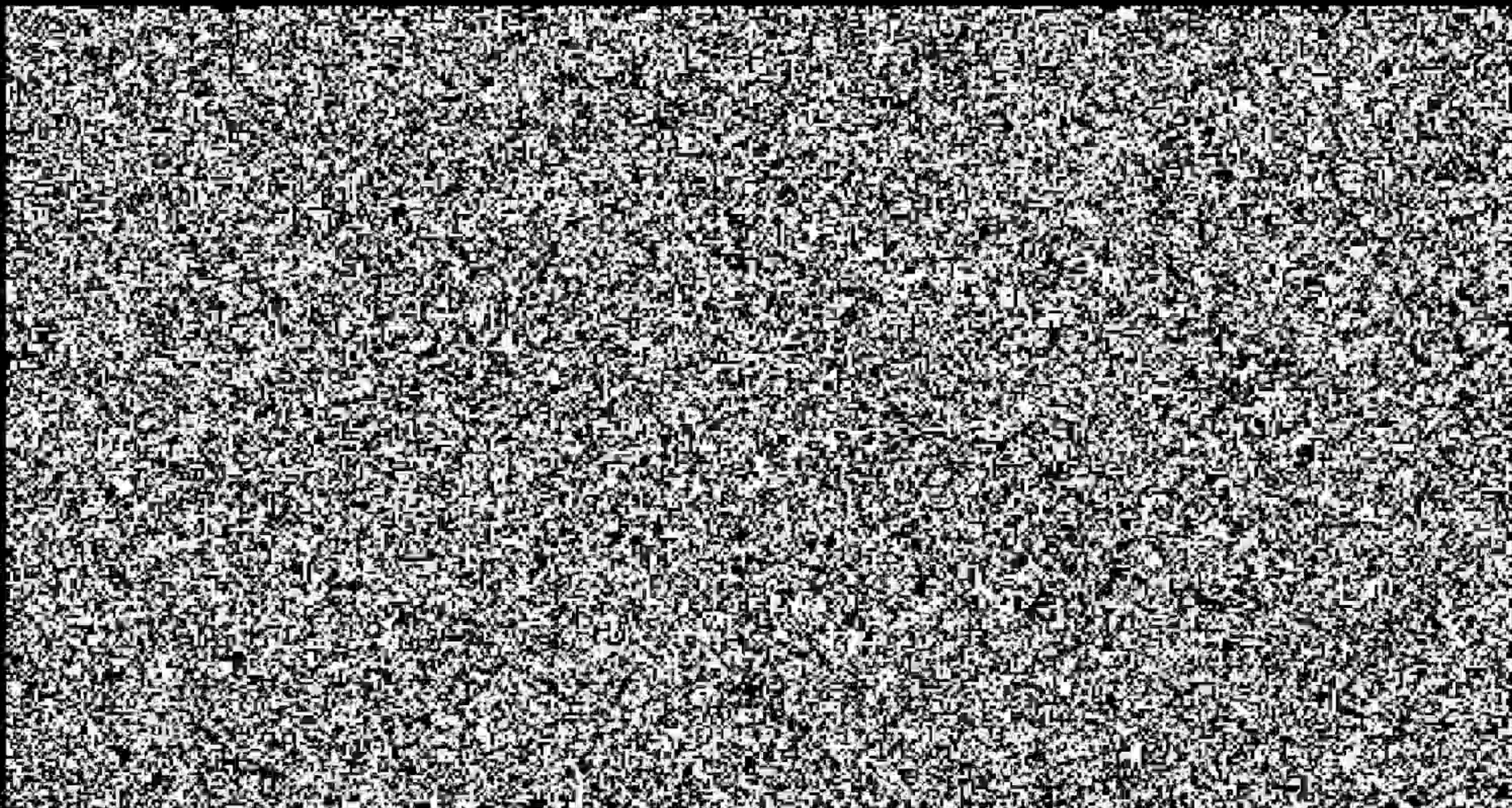
Marcella Noorman

Sashank Pisupati

Satpreet Singh

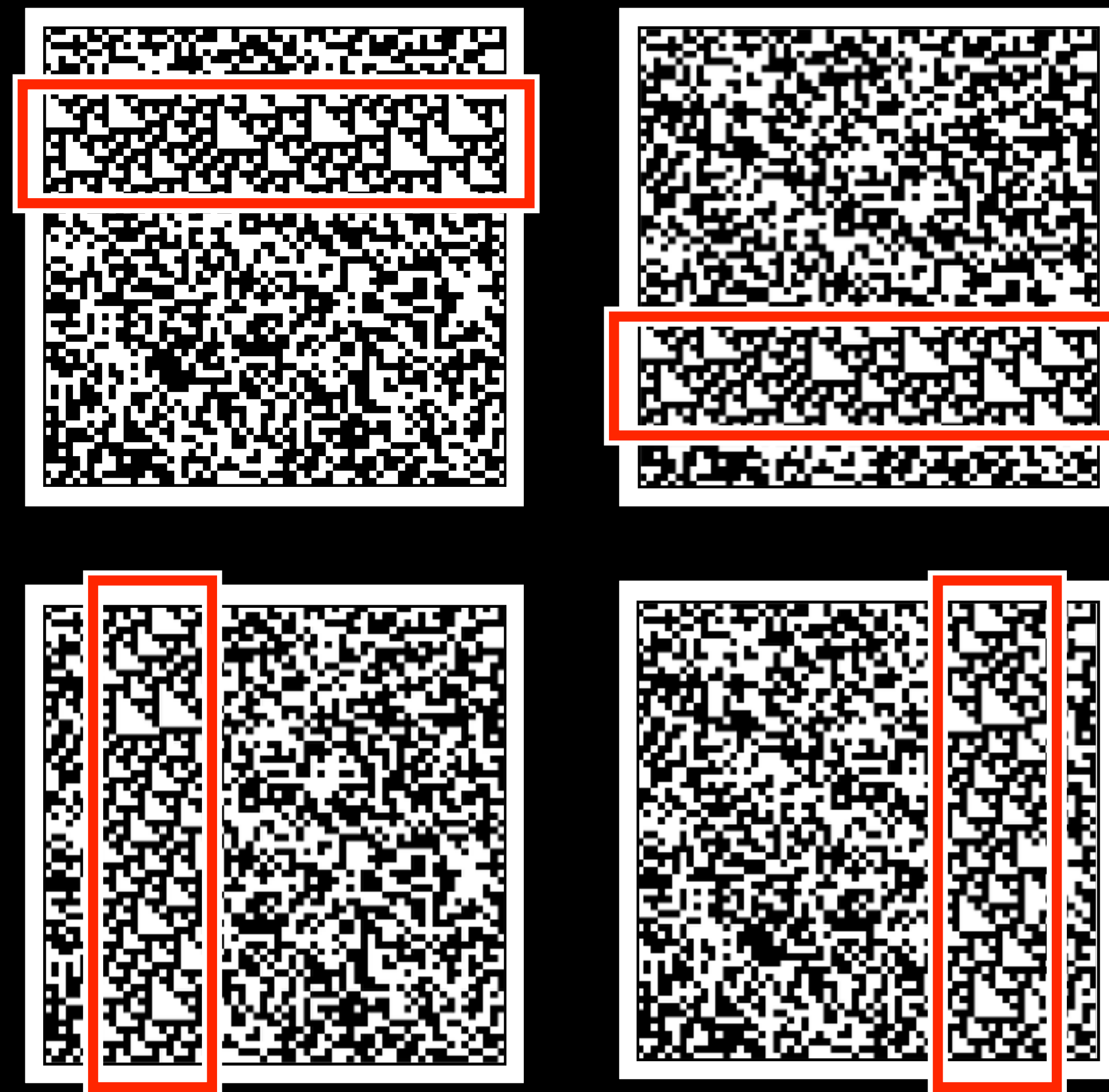
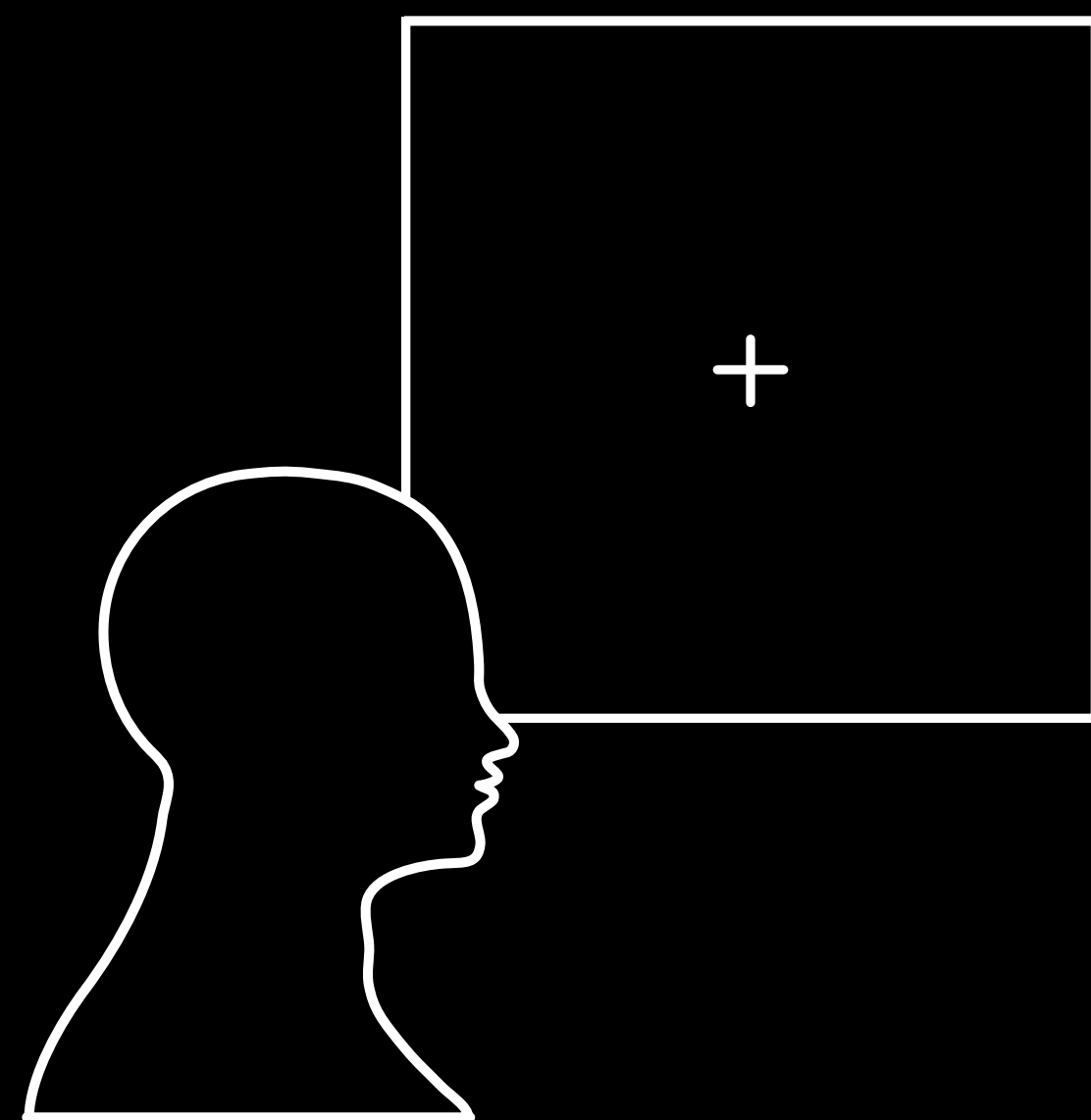
Charline Tessereau

A wing would be a most mystifying structure if one did not know that birds flew. One might observe that it could be extended a considerable distance, that it had a smooth covering of feathers with conspicuous markings, that it was operated by powerful muscles, and that strength and lightness were prominent features of its construction. These are important facts, but by themselves they do not tell us that birds fly. Yet without knowing this, and without understanding something of the principles of flight, a more detailed examination of the wing itself would probably be unrewarding.

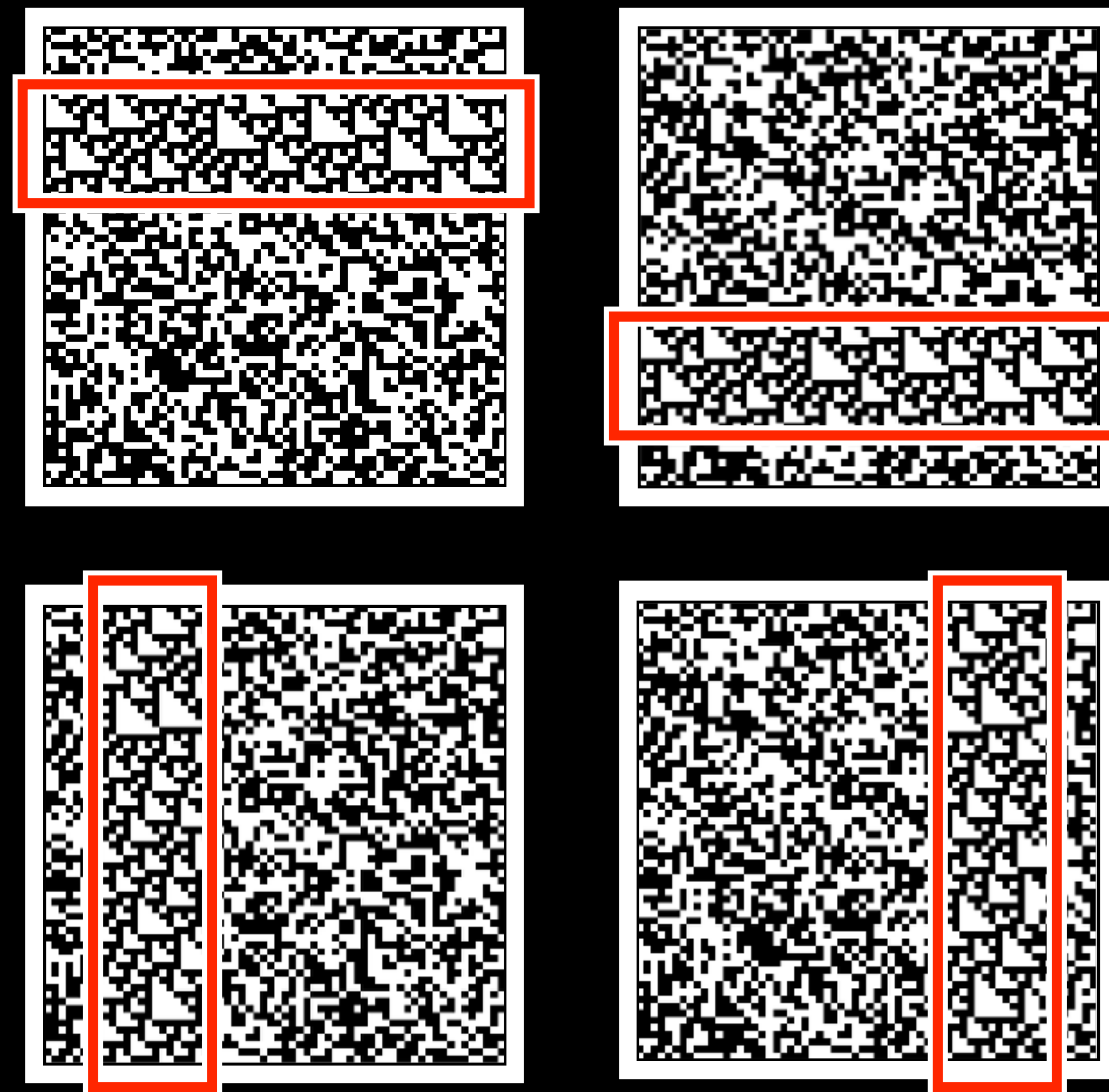
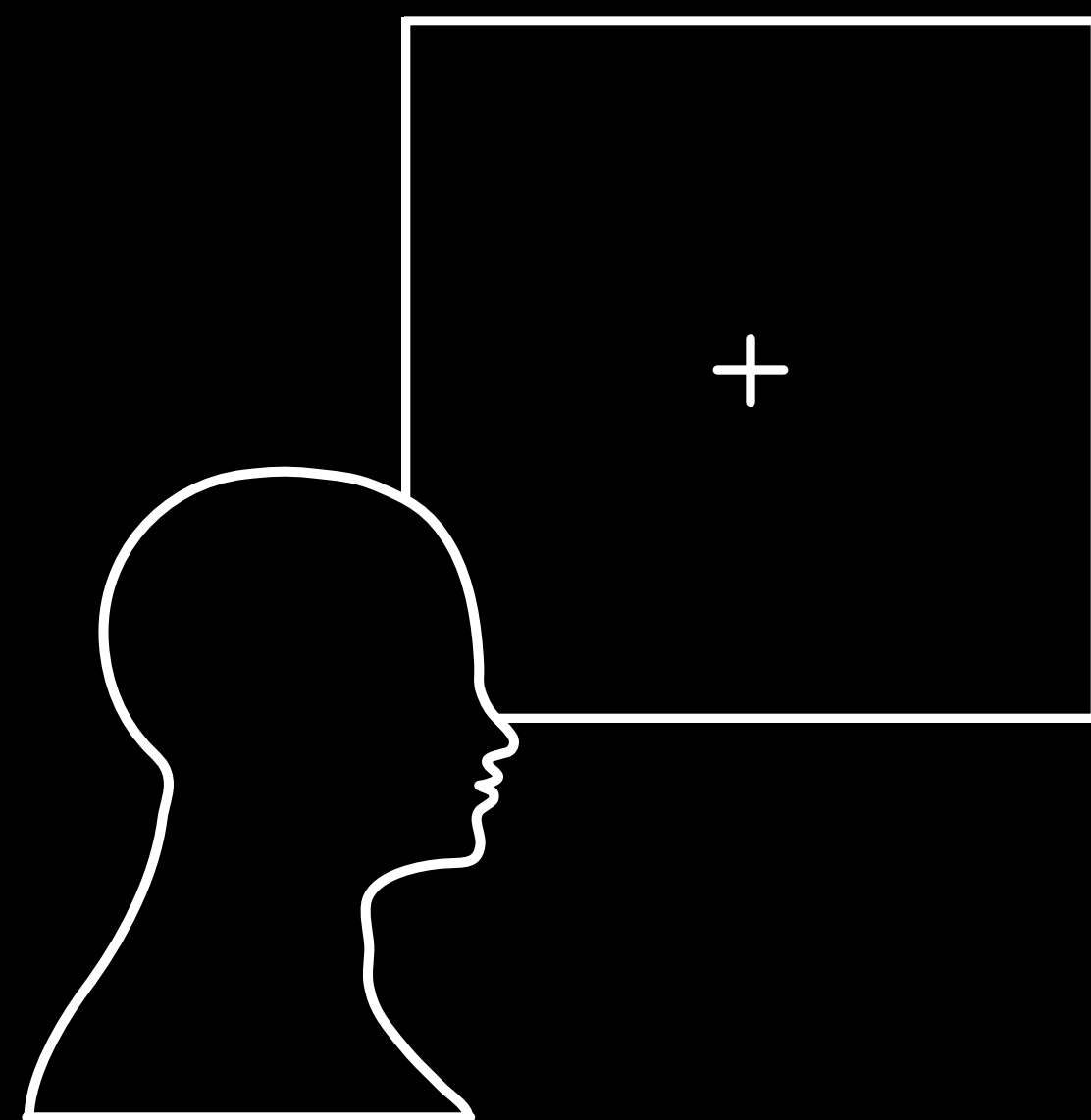




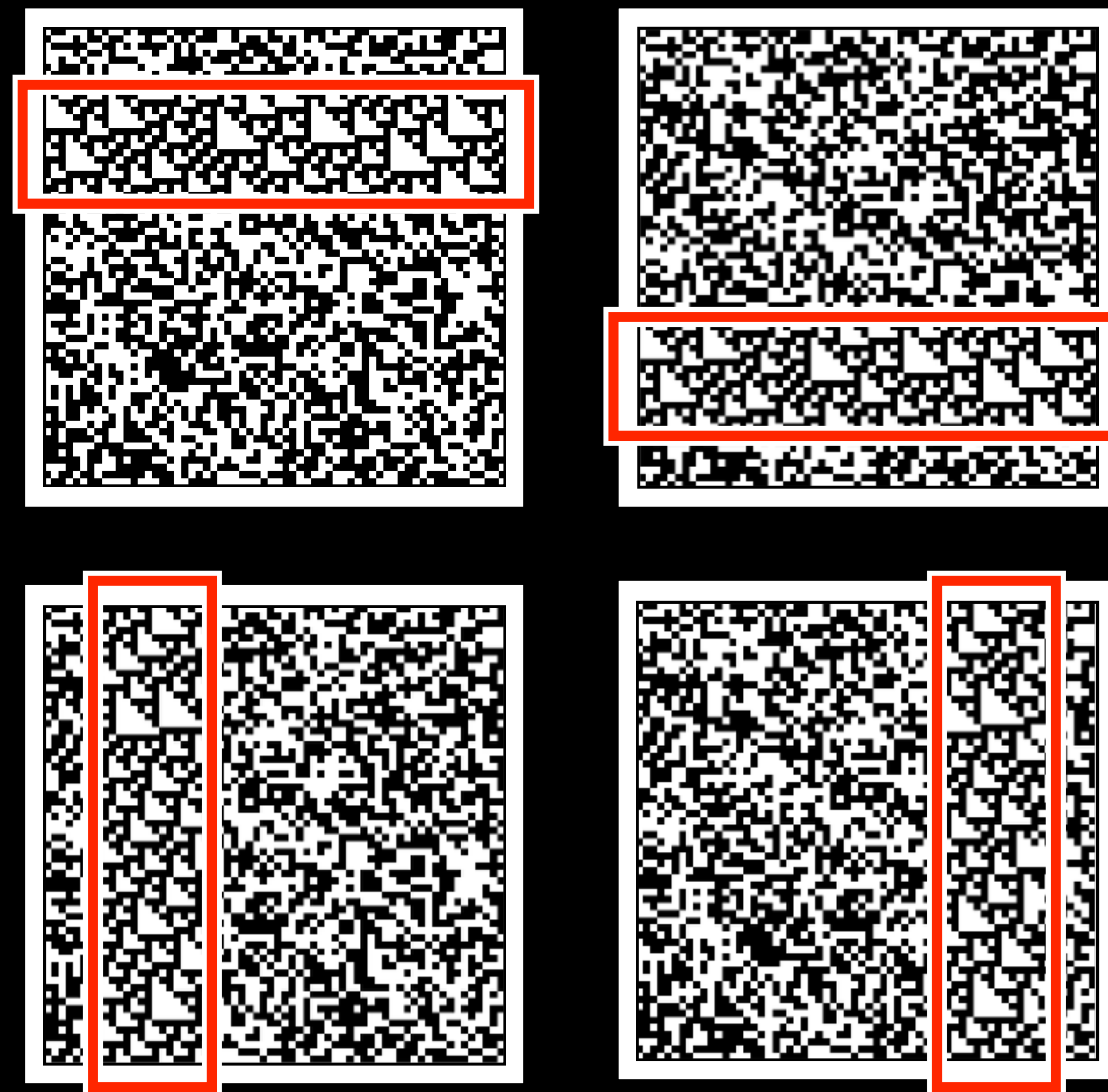
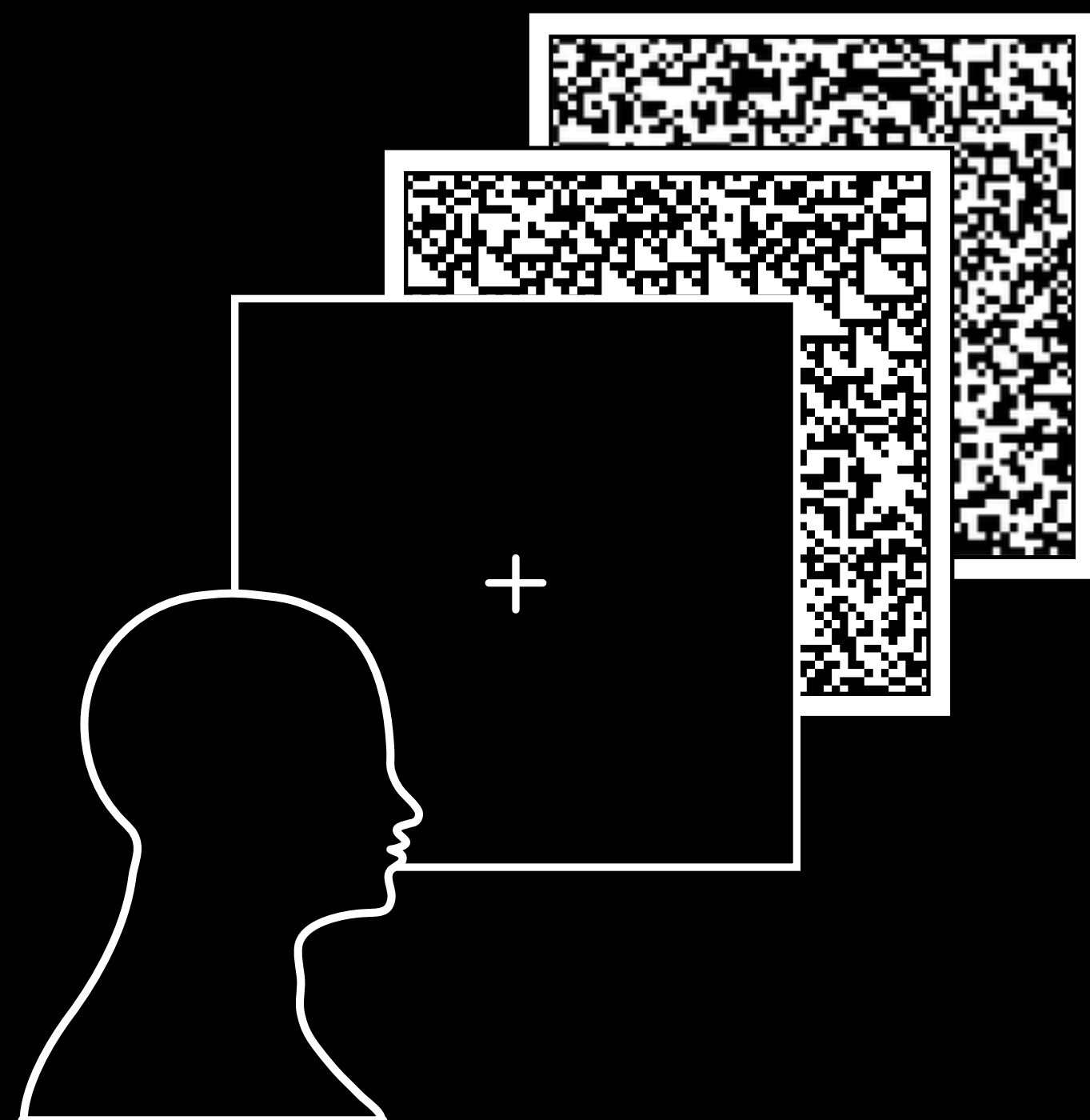
psychophysics & stimulus generation: Jonathan Victor, Mary Conte
Victor & Conte (2012), JOSA A; Victor, Thengone, & Conte (2013), J Vision



psychophysics & stimulus generation: Jonathan Victor, Mary Conte
Victor & Conte (2012), JOSA A; Victor, Thengone, & Conte (2013), J Vision



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Victor & Conte (2012), JOSA A; Victor, Thengone, & Conte (2013), J Vision

+







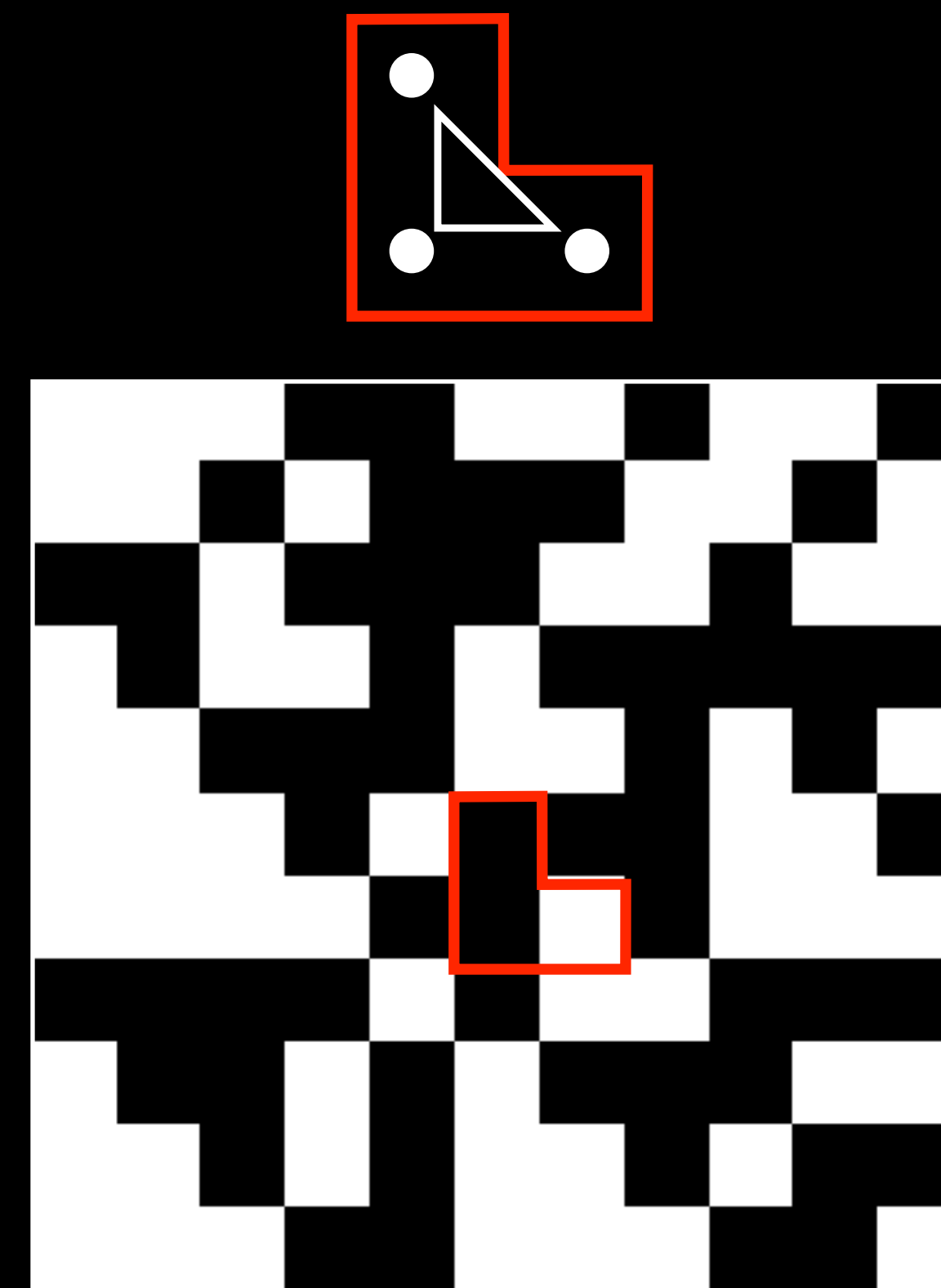
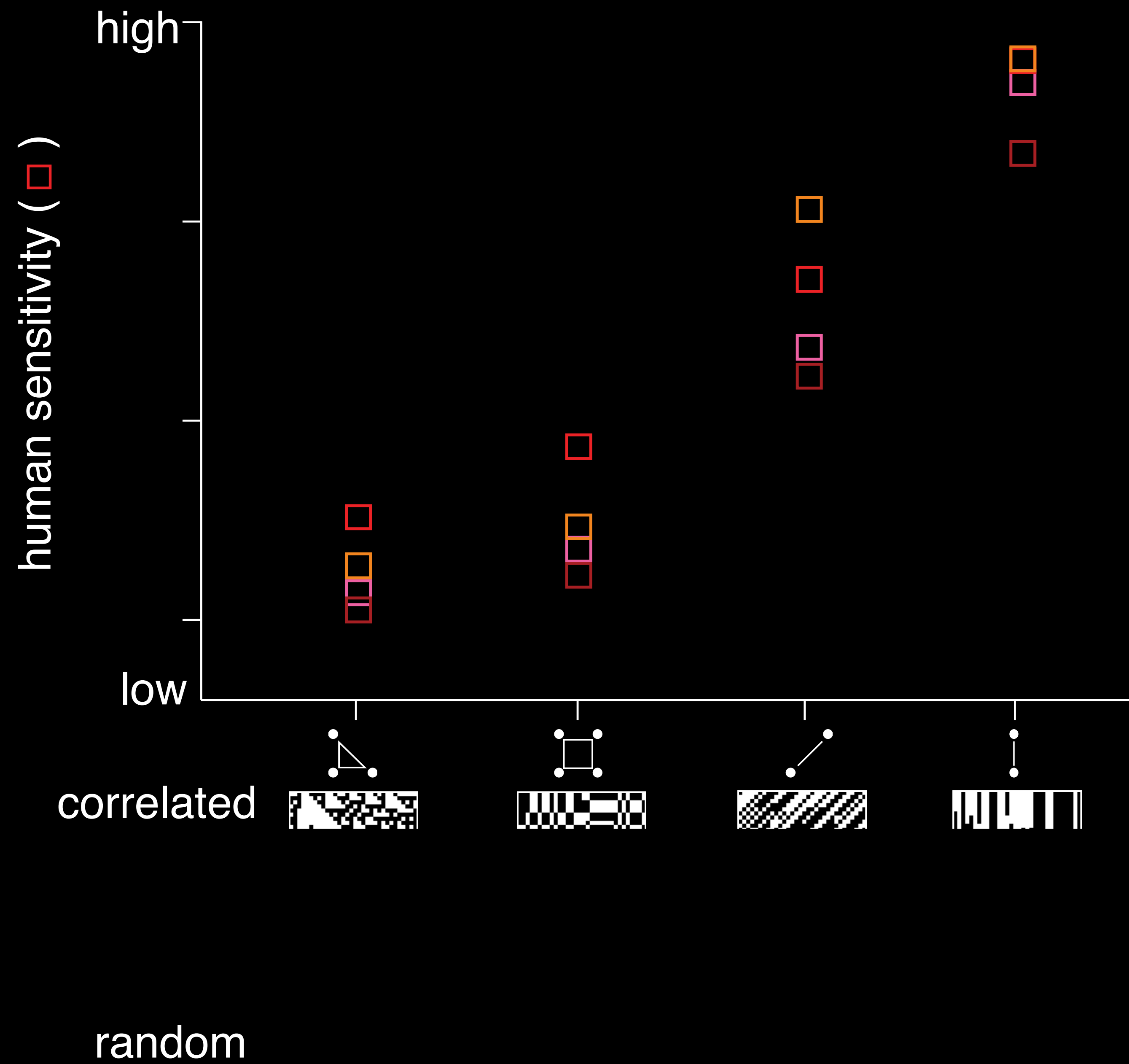
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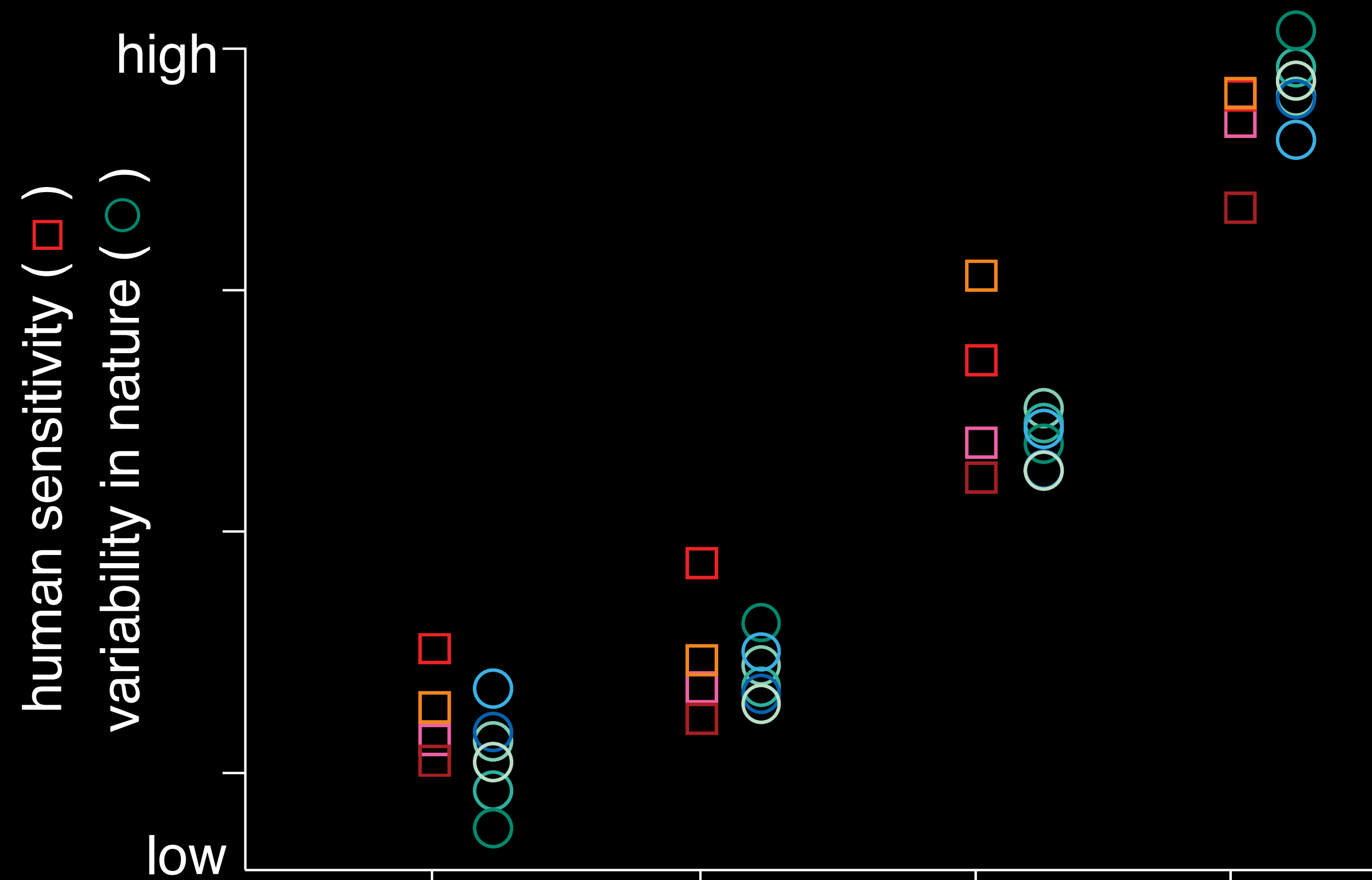






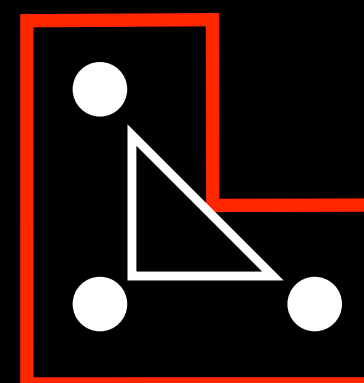
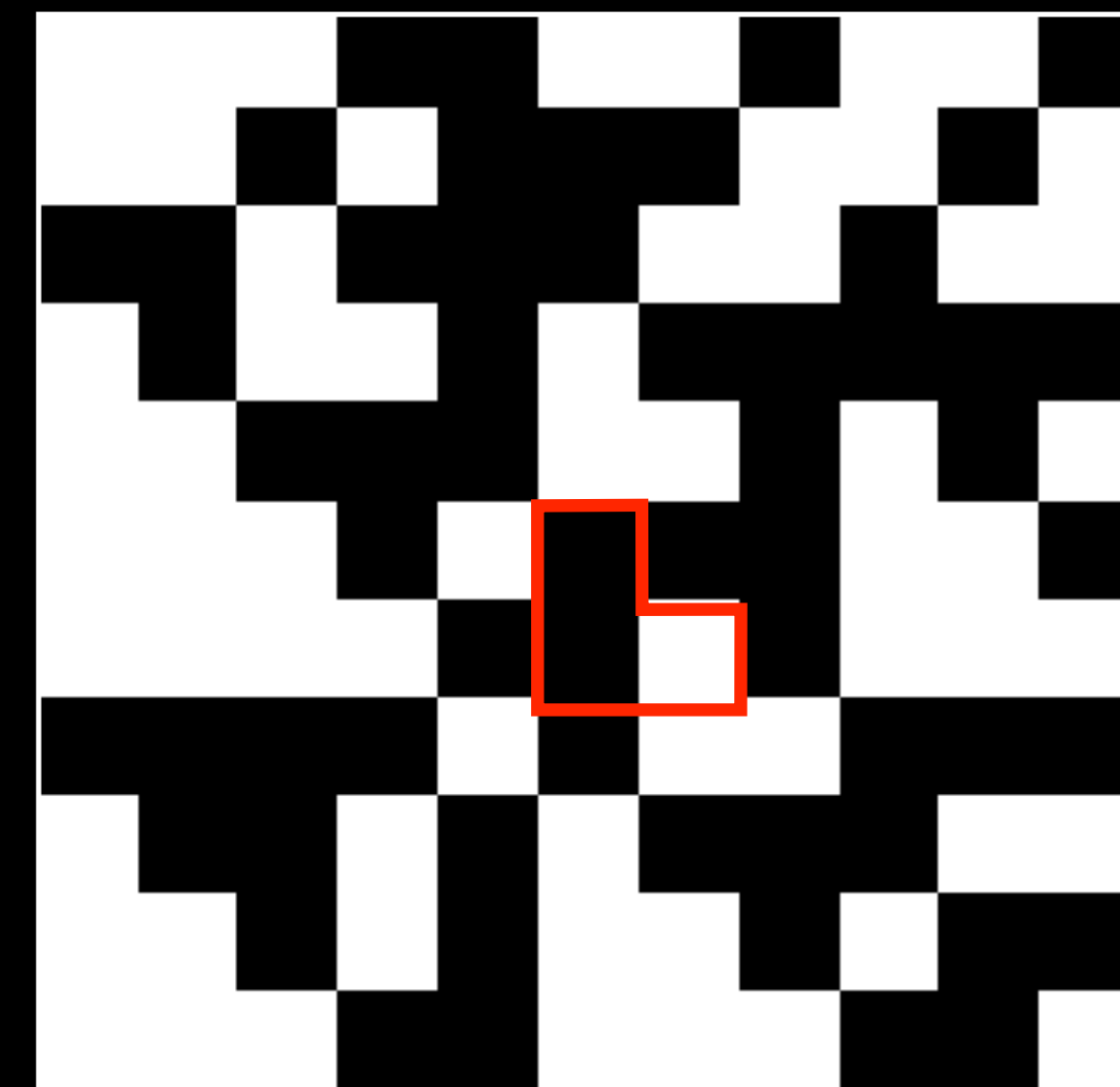
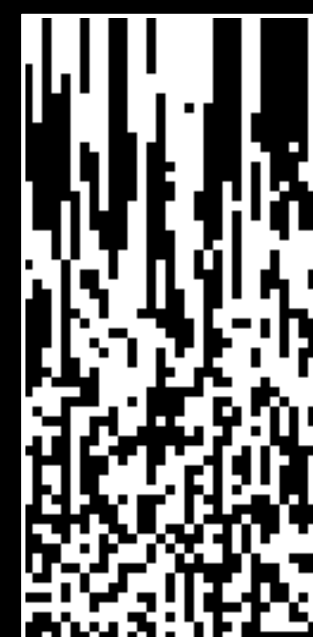






correlated

random



claim:

you should be more sensitive to visual features
that are more variable

...because they are more informative

claim:

variable

you should be more sensitive to [^]visual features ...

performance advantage

...because they are more informative

lawfulness of the world

the world is lawful

animals & brains can exploit the *lawfulness of the world*
to achieve a *performance advantage*

how might a system do this?

reduce redundancy / build compact representations

combat noise / correct errors

resolve ambiguities / reduce uncertainty

make predictions / improve future performance

the normative approach

lawfulness of the world
performance of a system



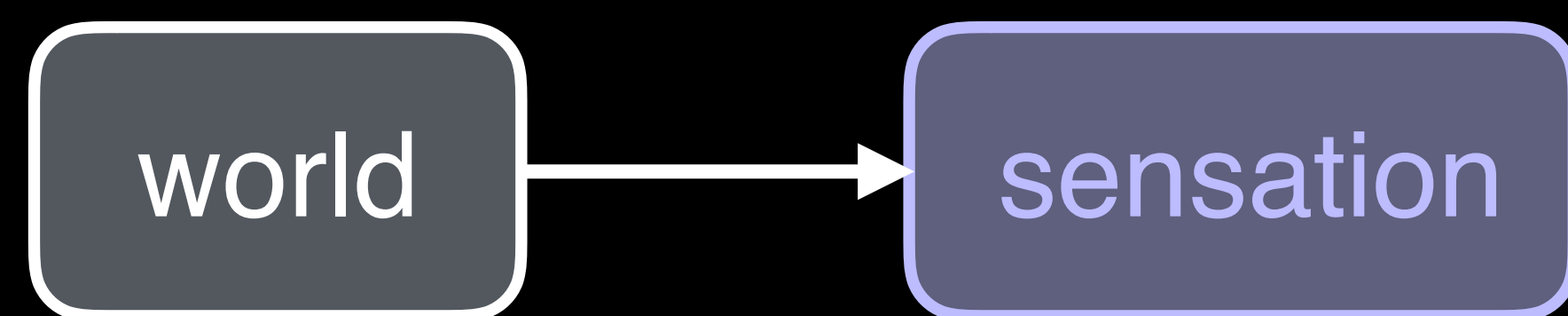
specify:

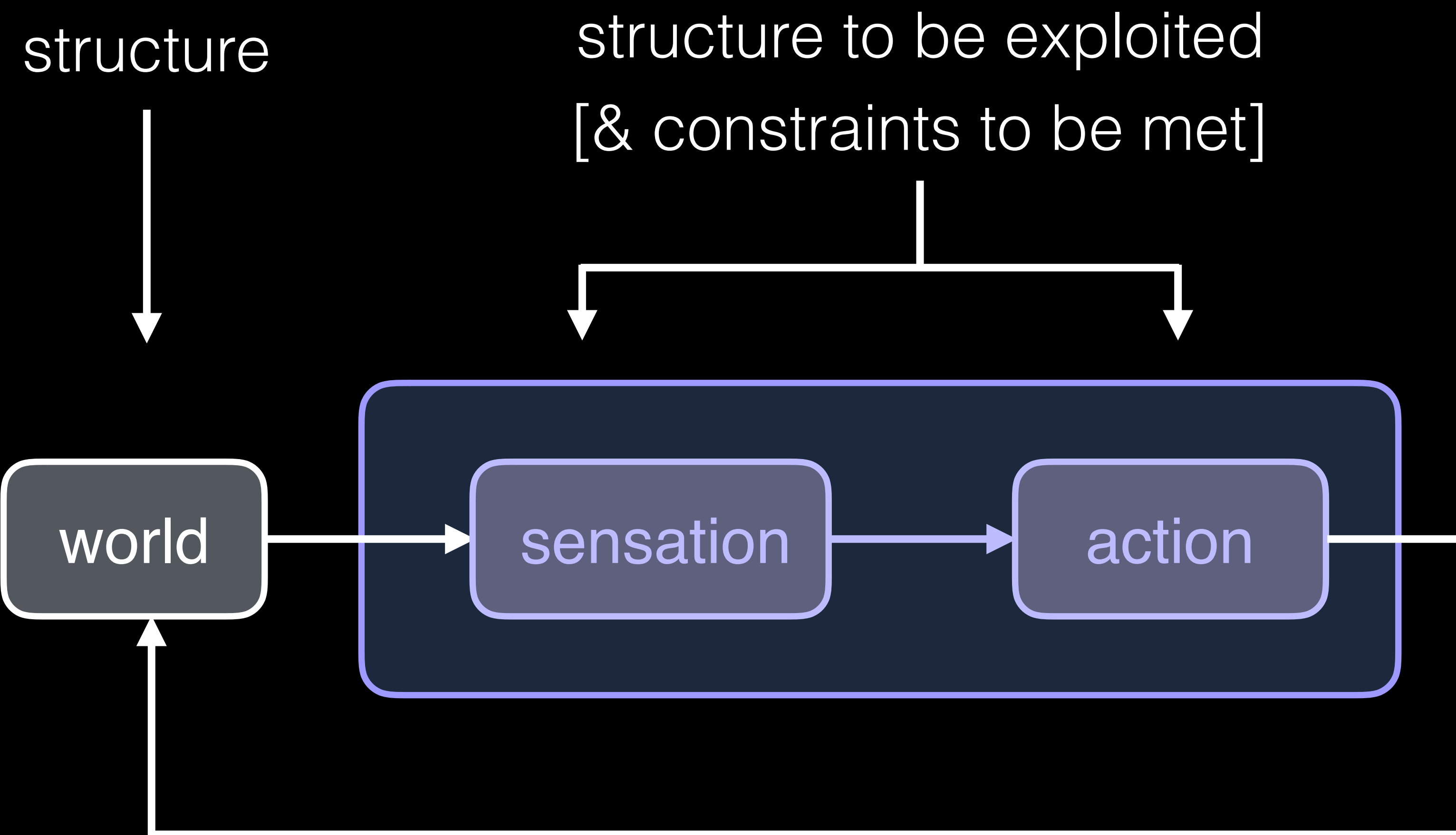
function to be performed [maximizing information about patterns]

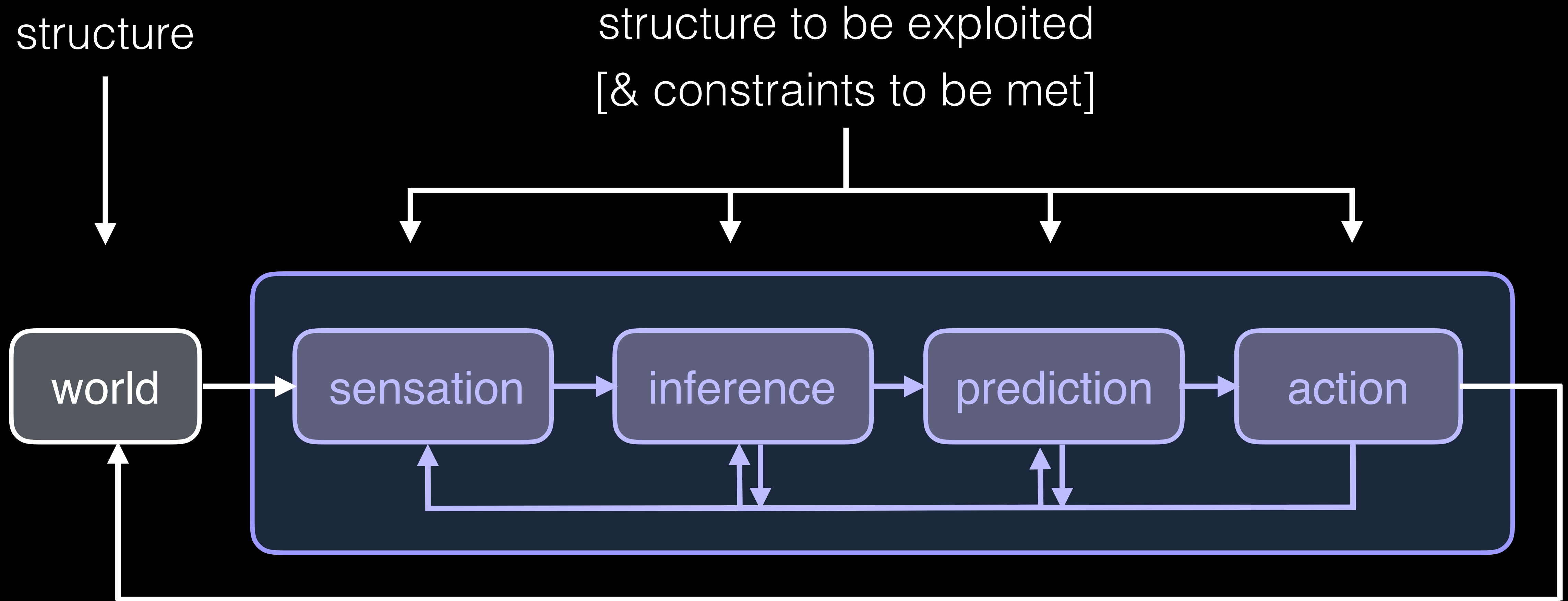
context in which function will be performed [natural visual world]

constraints on the system that performs this function [bandwidth &
[precision, accuracy, speed, energy, ...] noise constraints]

determine **best solution** for achieving particular function [tune sensitivity
in particular context & subject to particular constraints to variability]







PART 1

remove redundancy
combat noise

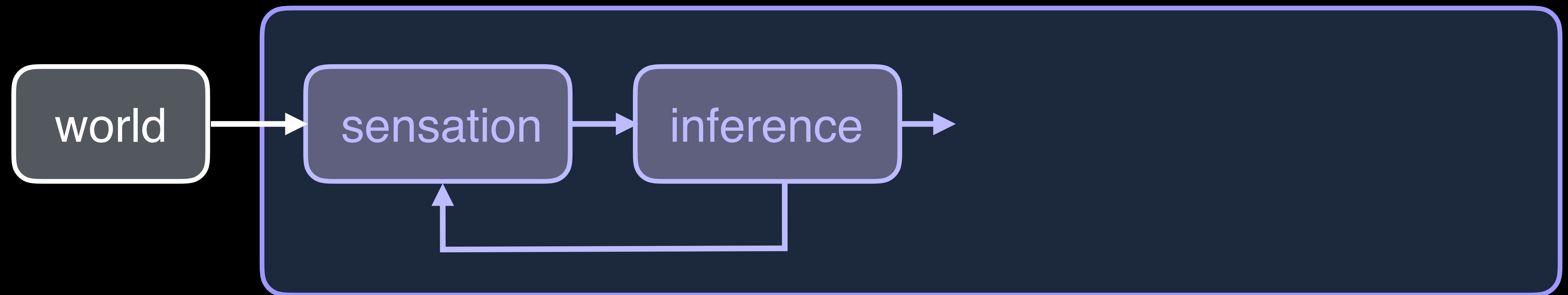


PART 1

remove redundancy
combat noise

PART 2

resolve ambiguity



PART 1

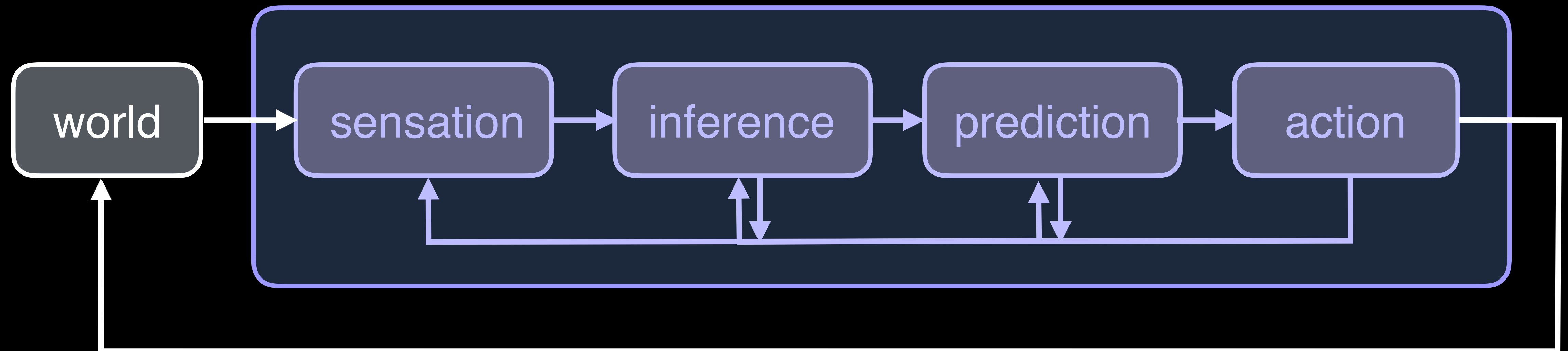
remove redundancy
combat noise

PART 2

resolve ambiguity

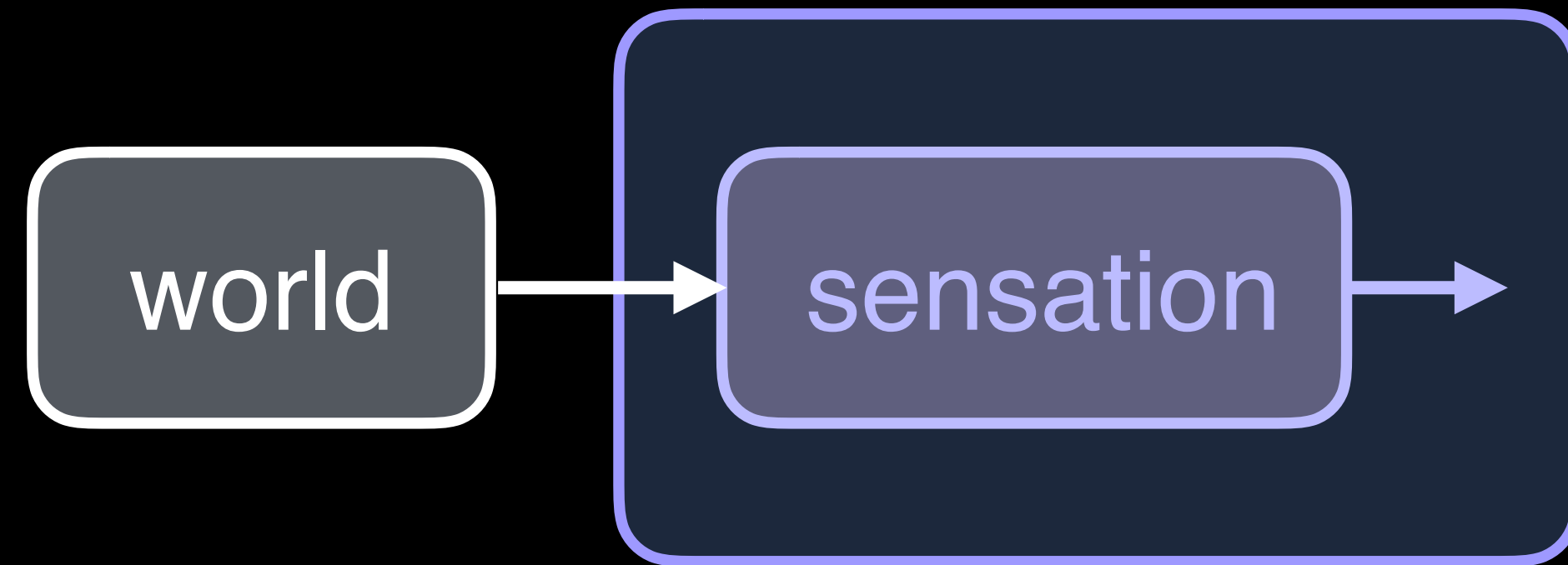
PART 3

make & use
predictions



PART 1

remove redundancy
combat noise



When we begin to consider perception as an information-handling process, it quickly becomes clear that much of the information received by any higher organism is *redundant*.

[this means that] if we know at a given moment the states of a limited number of receptors (i.e., whether they are firing or not firing), we can make better-than-chance inferences with respect to the prior and subsequent states of these receptors, and also with respect to the present, prior, and subsequent states of other receptors.

[this is] precisely equivalent to an assertion that the world as we know it is lawful.

It appears likely that a major function of the perceptual machinery is to strip away some of the redundancy of stimulation, to describe or encode incoming information in a form more economical than that in which it impinges on the receptors.

Barlow's redundancy reduction hypothesis

goal: maximize information

$$I(R; S) = H(R) - H(R|S)$$

0 low input noise

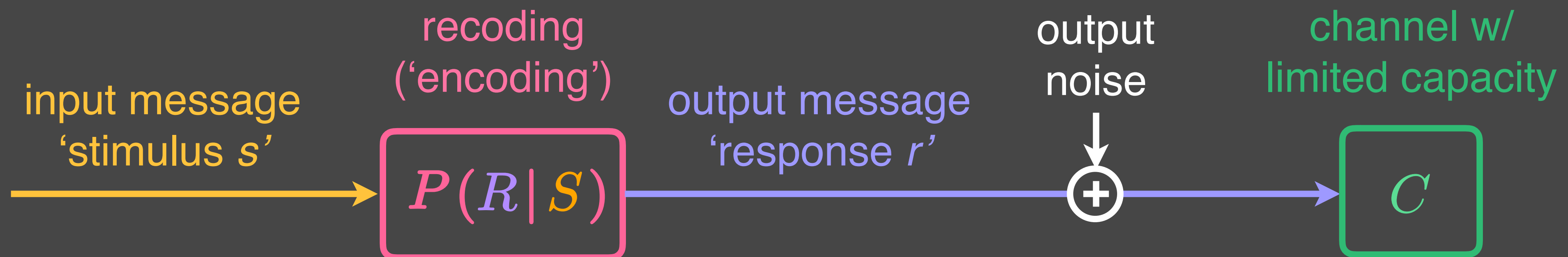
maximize response entropy

$$= H(R)$$

OR

minimize redundancy

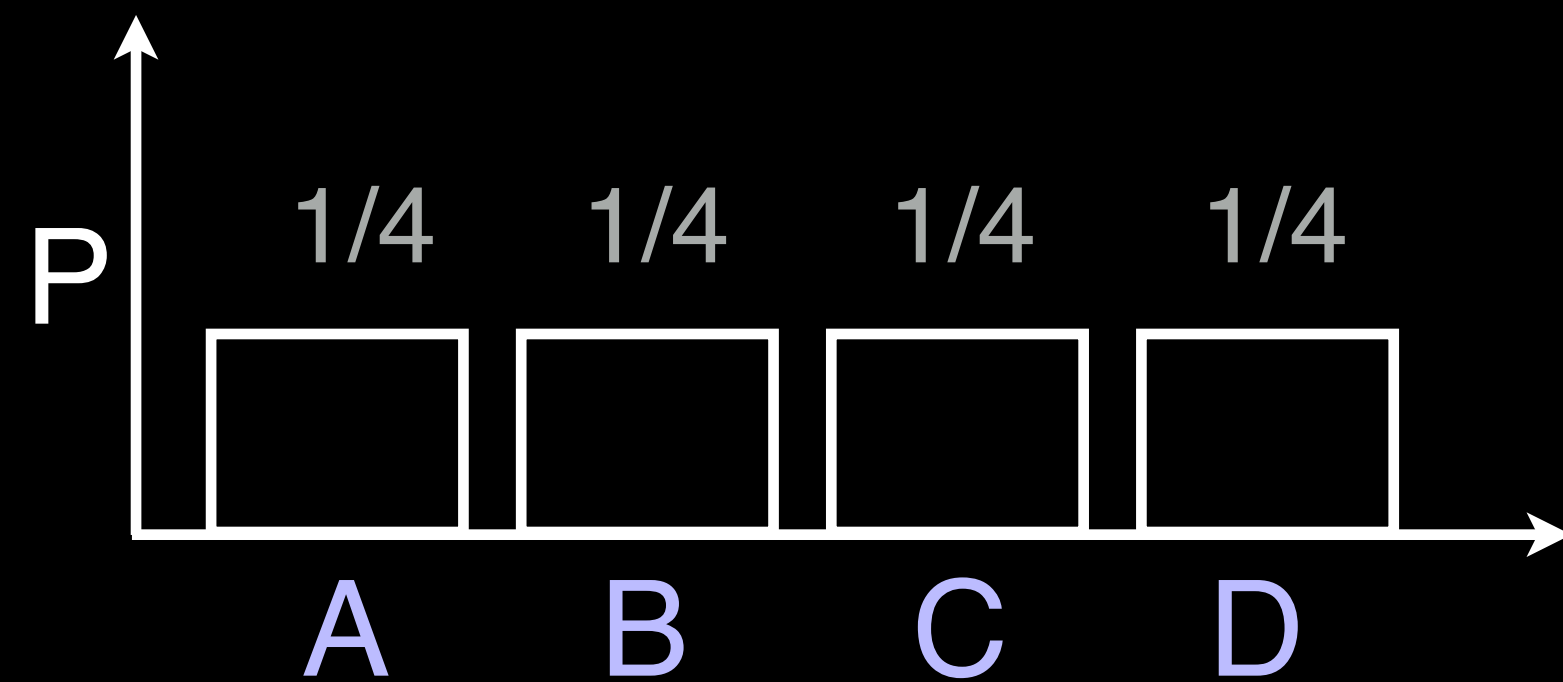
$$\mathcal{R} = 1 - H(R)/C$$



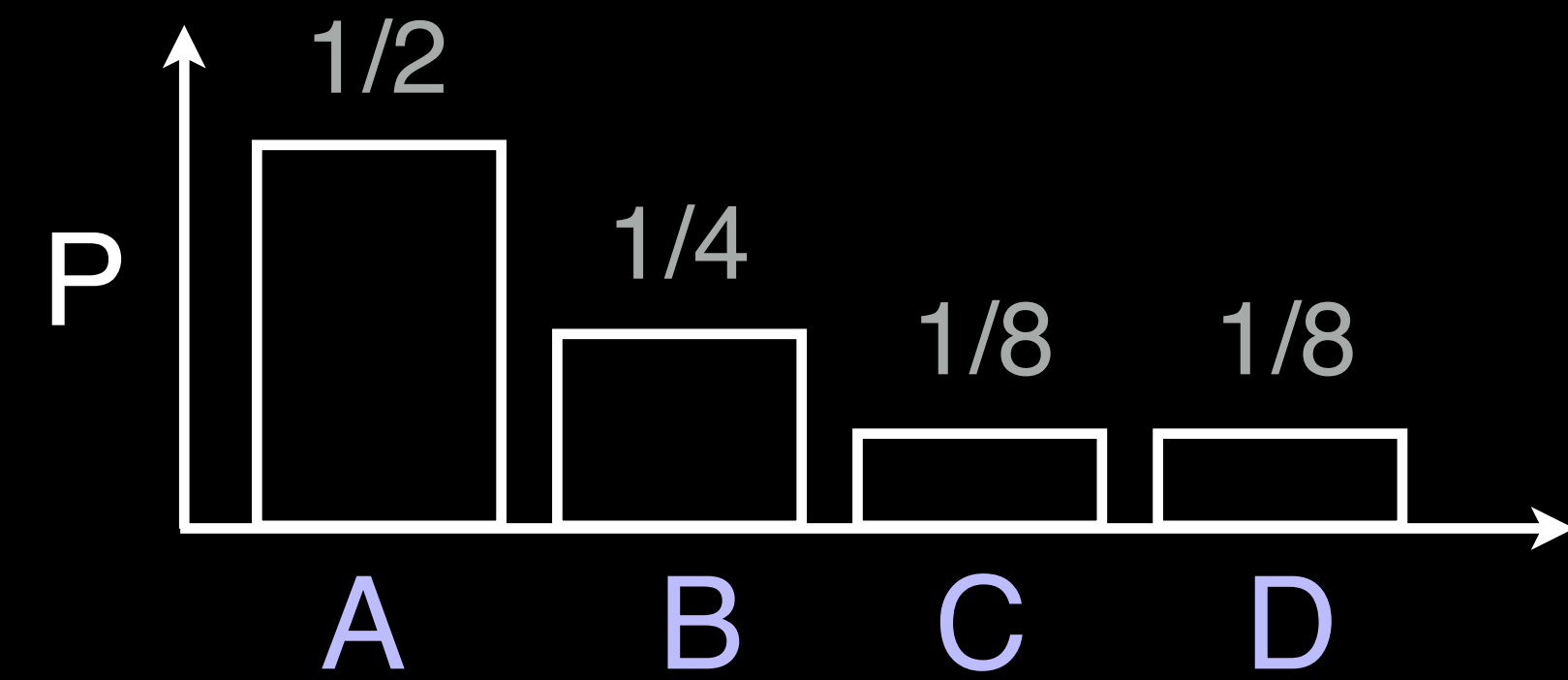
entropy $H(R)$

average # yes/no questions needed
to determine output with certainty

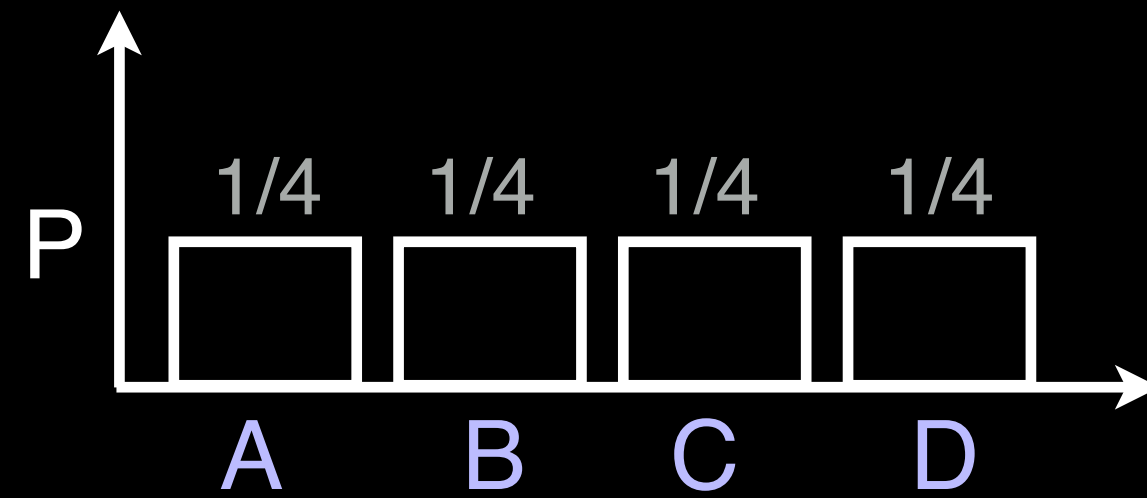
machine 1 →



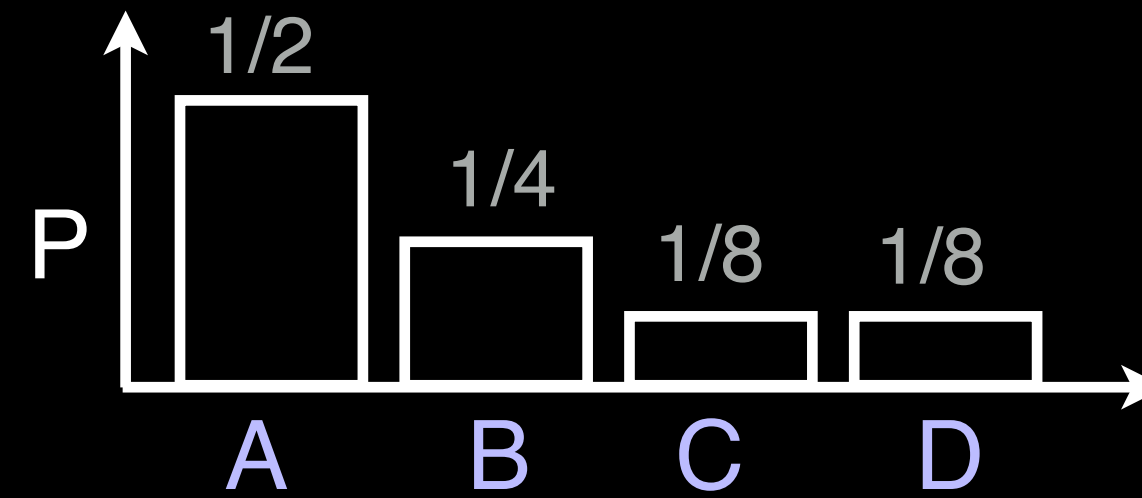
machine 2 →



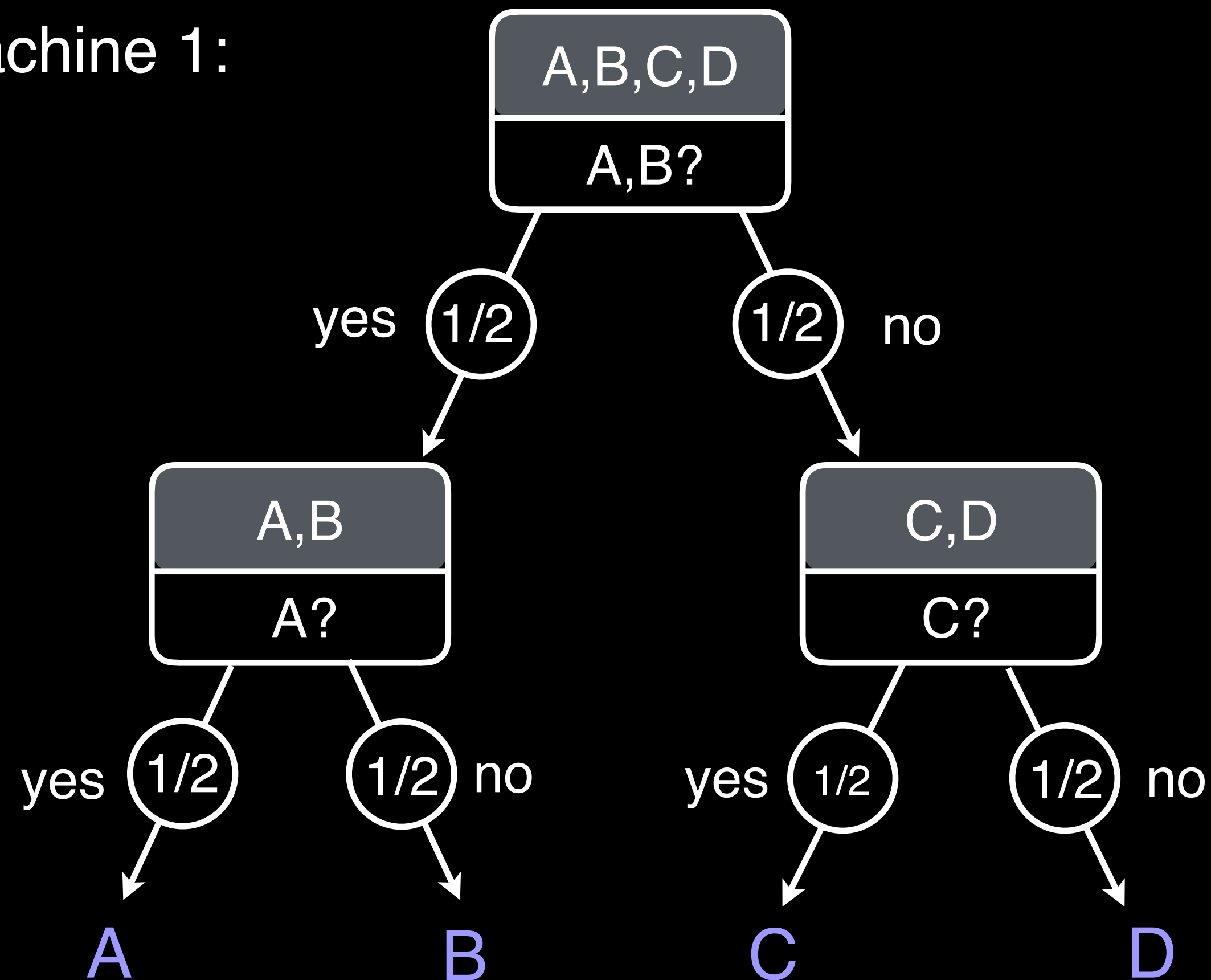
machine 1



machine 2



machine 1:



entropy

average # yes/no questions needed
to determine output with certainty

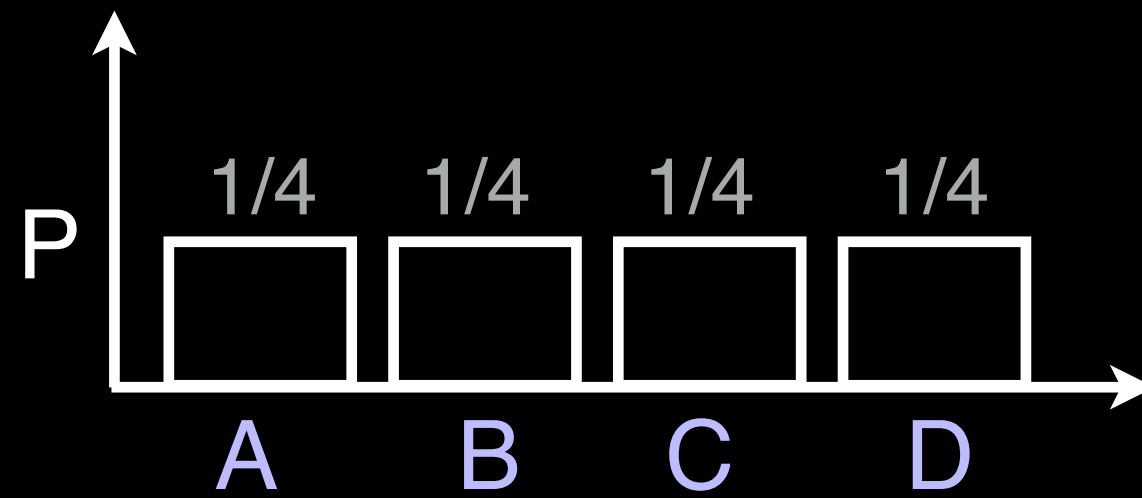
$$= \sum \text{# questions per option} * \text{probability of option}$$

$$= \#Q_A * P_A + \#Q_B * P_B + \#Q_C * P_C + \#Q_D * P_D$$

$$\begin{aligned} H_1 &= 2 * (1/4) + 2 * (1/4) + 2 * (1/4) + 2 * (1/4) \\ &= 2 \text{ [bits]} \end{aligned}$$

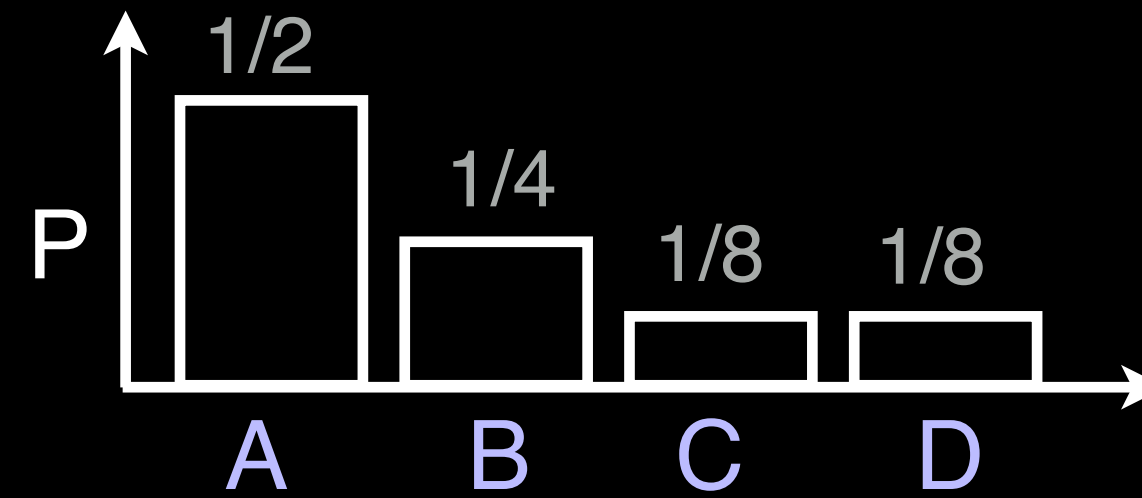
$$H_2 = ?$$

machine 1



H = 2 bits

machine 2



H = 1.75 bits

entropy

average # yes/no questions needed
to determine output with certainty

$$= \sum \text{# questions per option} * \text{probability of option}$$

$$= \#Q_A * P_A + \#Q_B * P_B + \#Q_C * P_C + \#Q_D * P_D$$

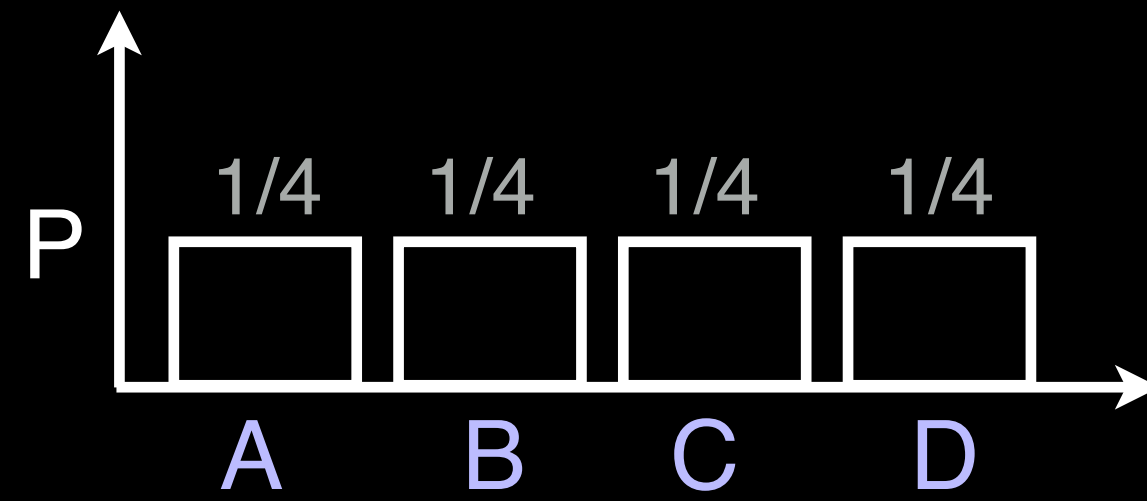
$$\begin{aligned} H_1 &= 2 * (1/4) + 2 * (1/4) + 2 * (1/4) + 2 * (1/4) \\ &= 2 \text{ [bits]} \end{aligned}$$

$$H_2 = 1.75 \text{ [bits]}$$

$$\# Q = -\log_2 P$$

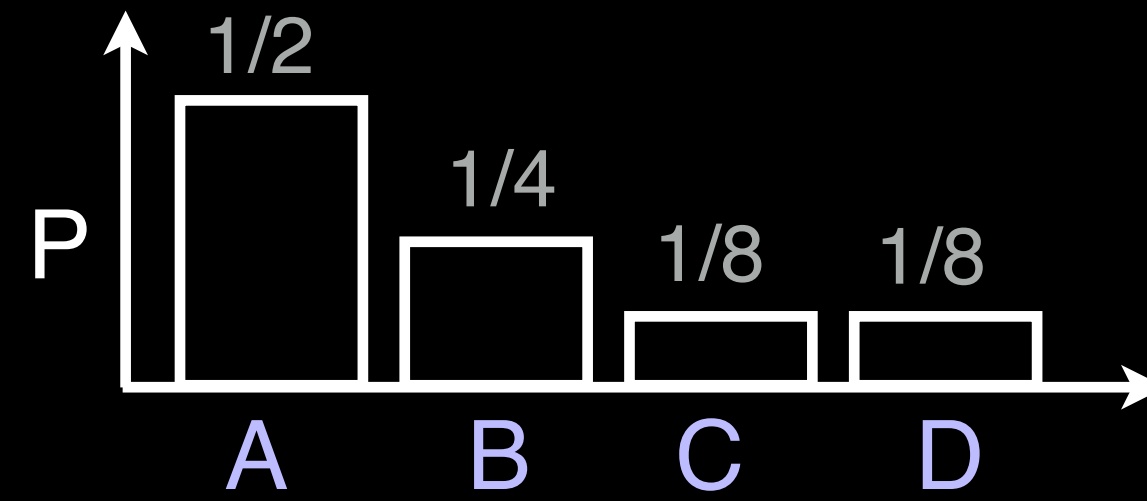
$$H = -\sum P \log_2 P$$

machine 1



$H = 2$ bits

machine 2



$H = 1.75$ bits

entropy

average # yes/no questions needed
to determine output with certainty

higher entropy



more uncertainty



greater reduction in uncertainty
by making a measurement

“classic” efficient coding hypothesis
(low input noise)

goal: maximize information

$$I(R; S) = H(R) - H(R|S)$$

0 low input noise

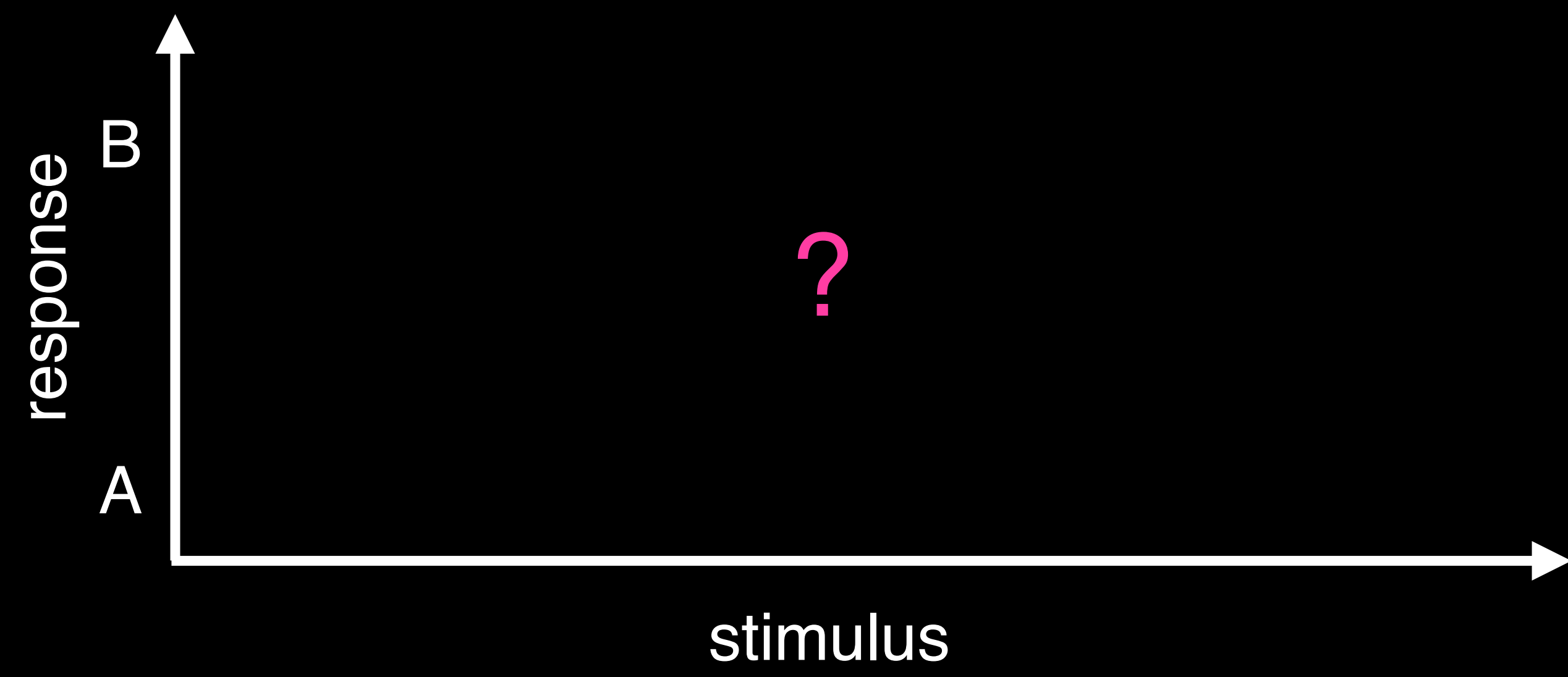
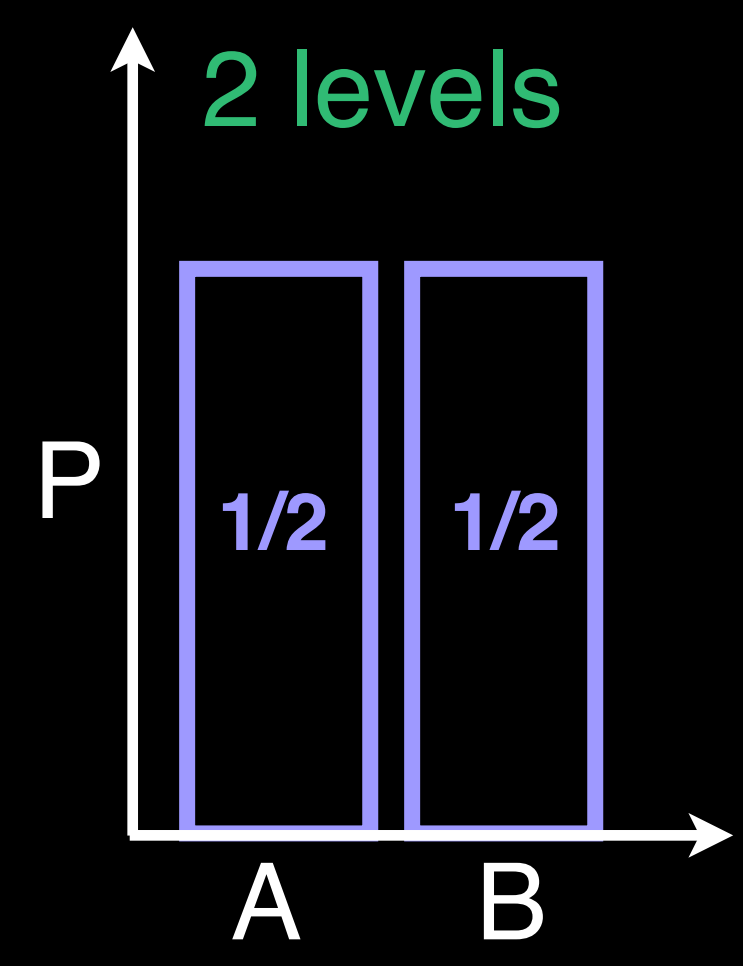
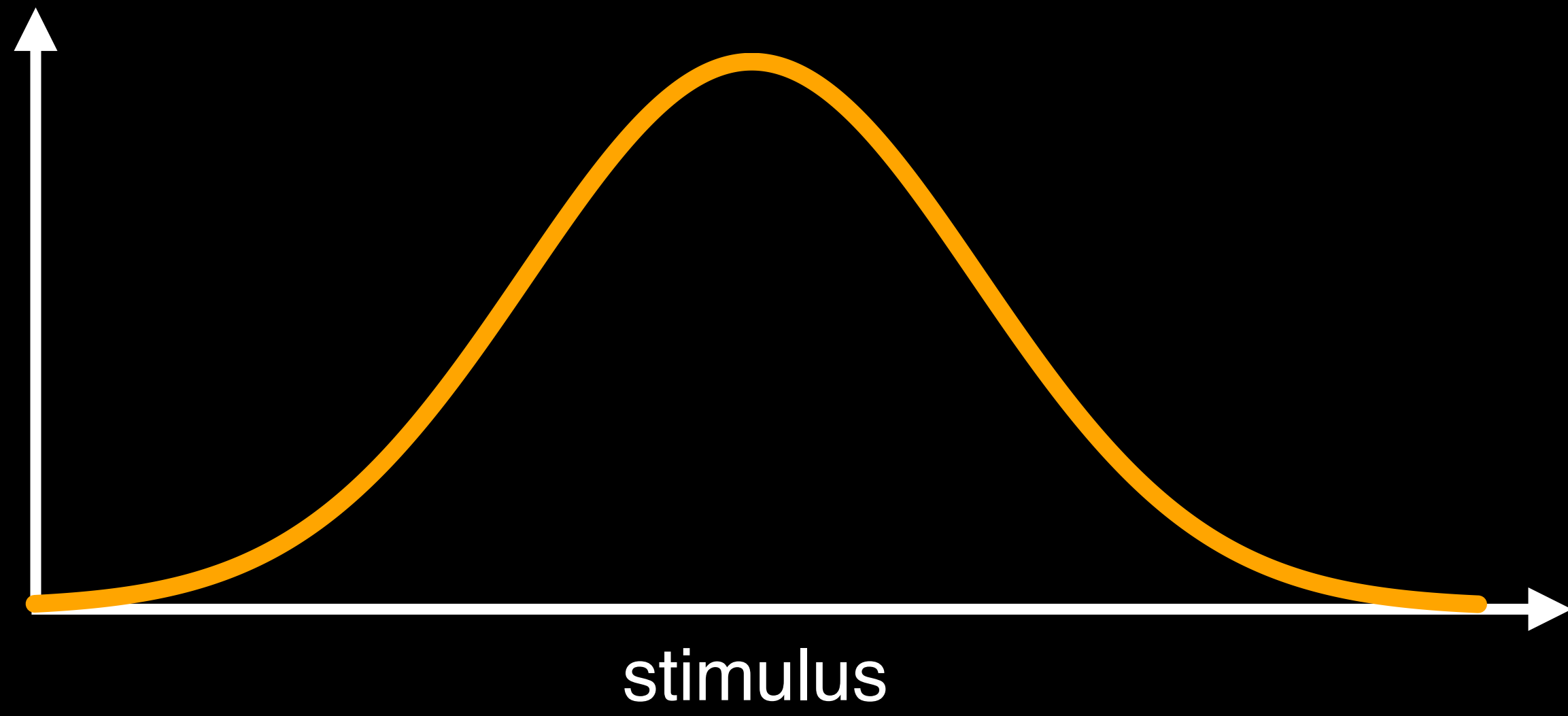
maximize response entropy

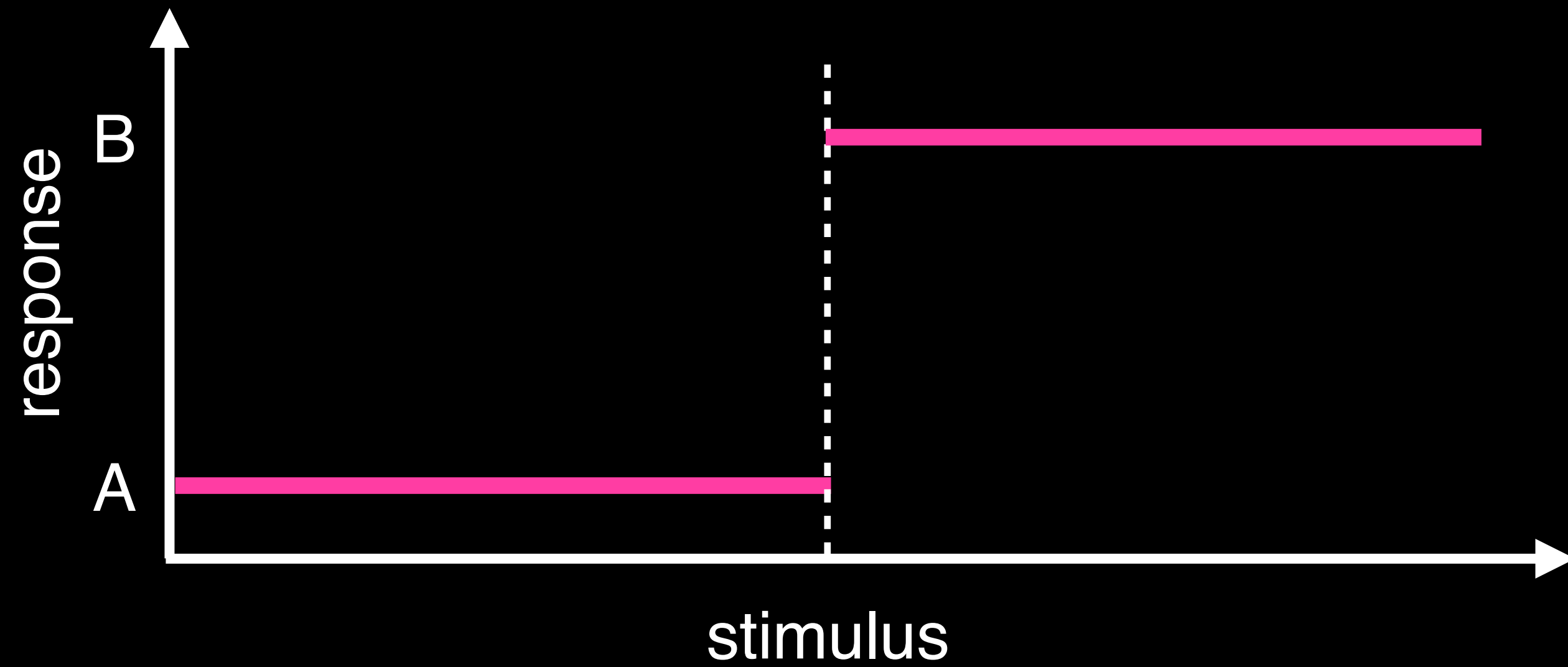
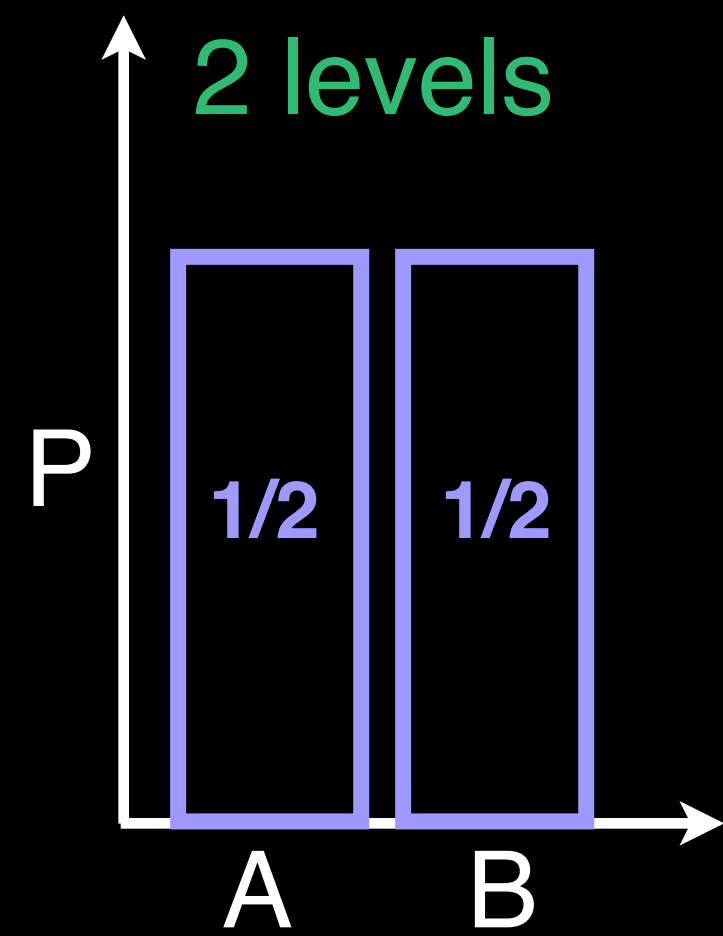
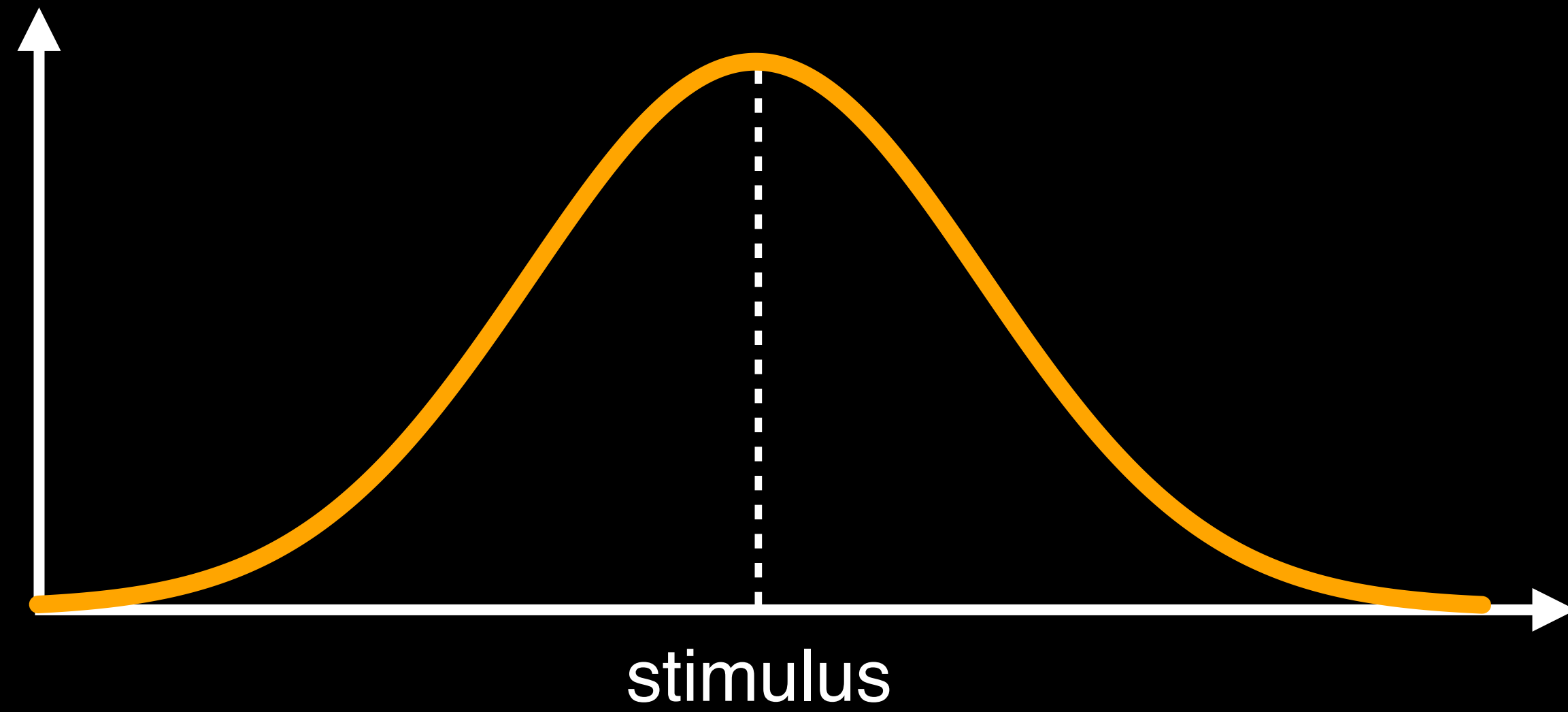
$$= H(R)$$

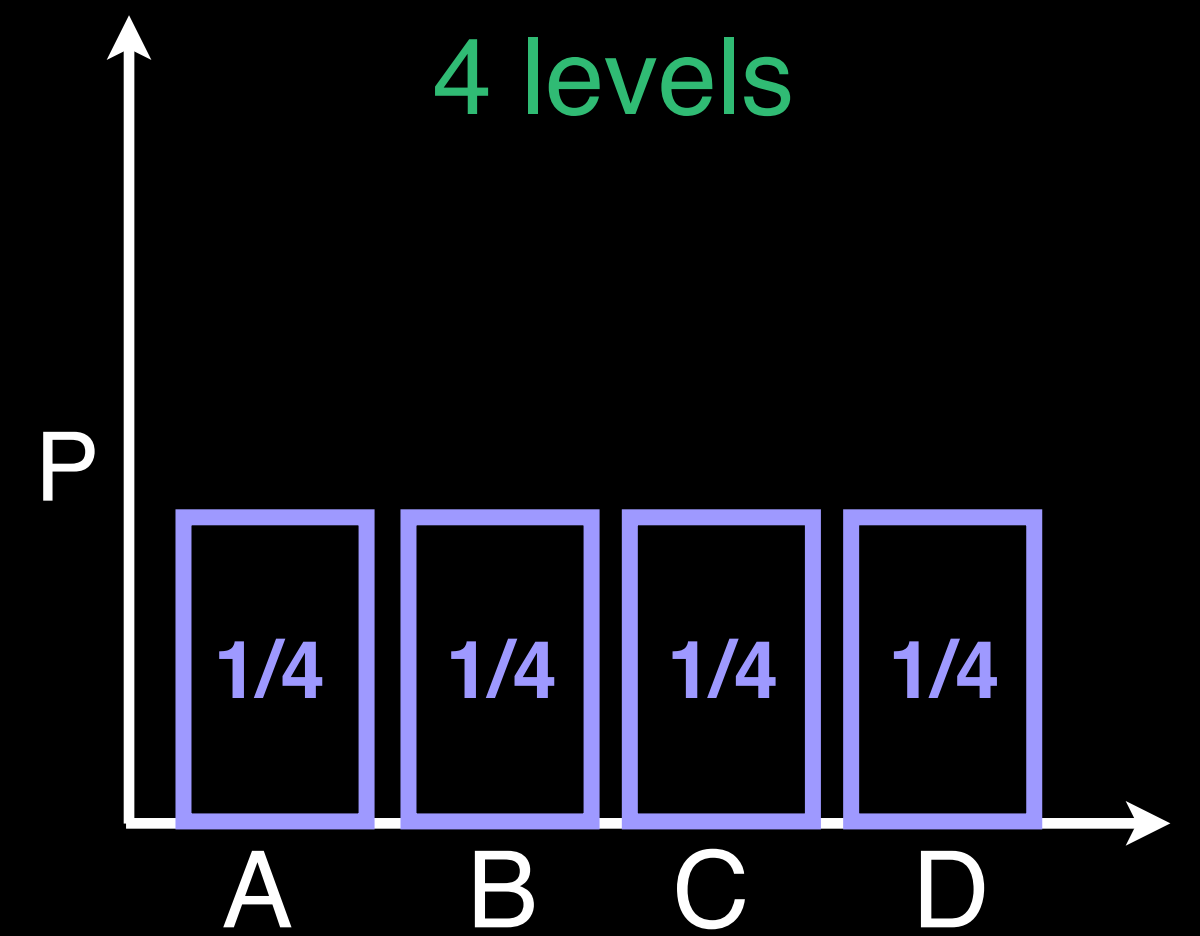
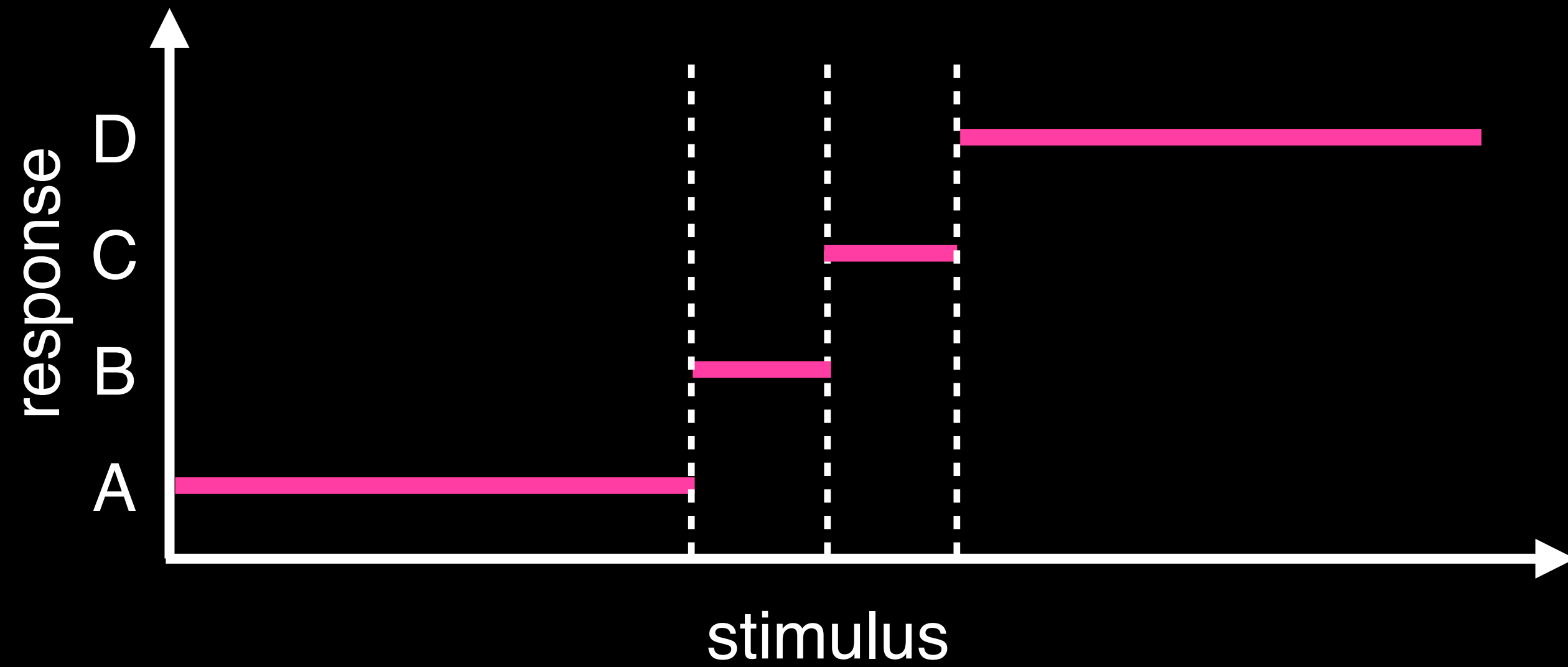
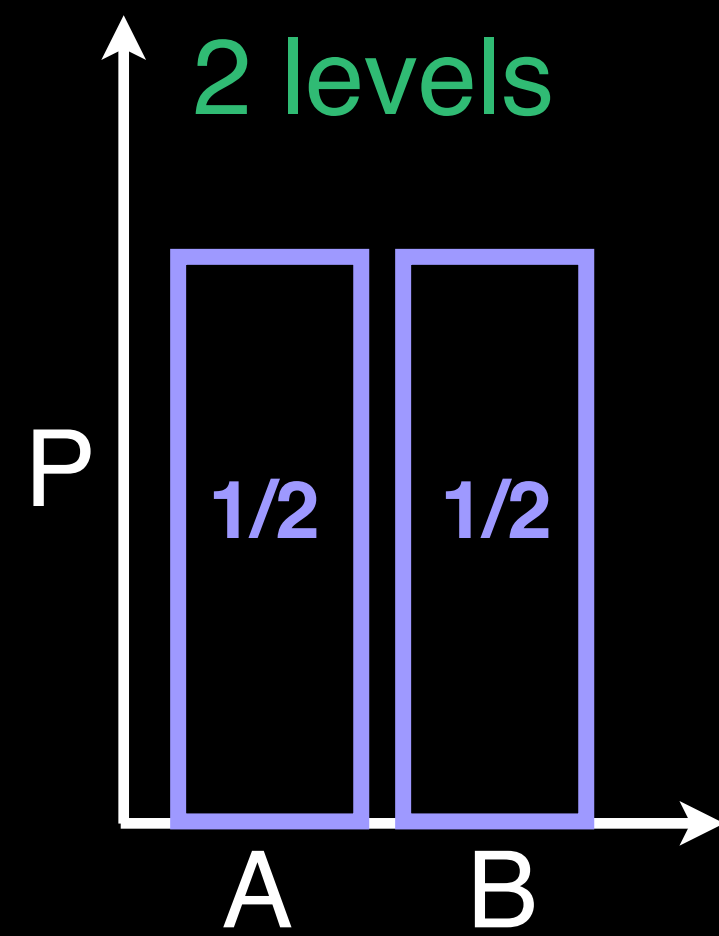
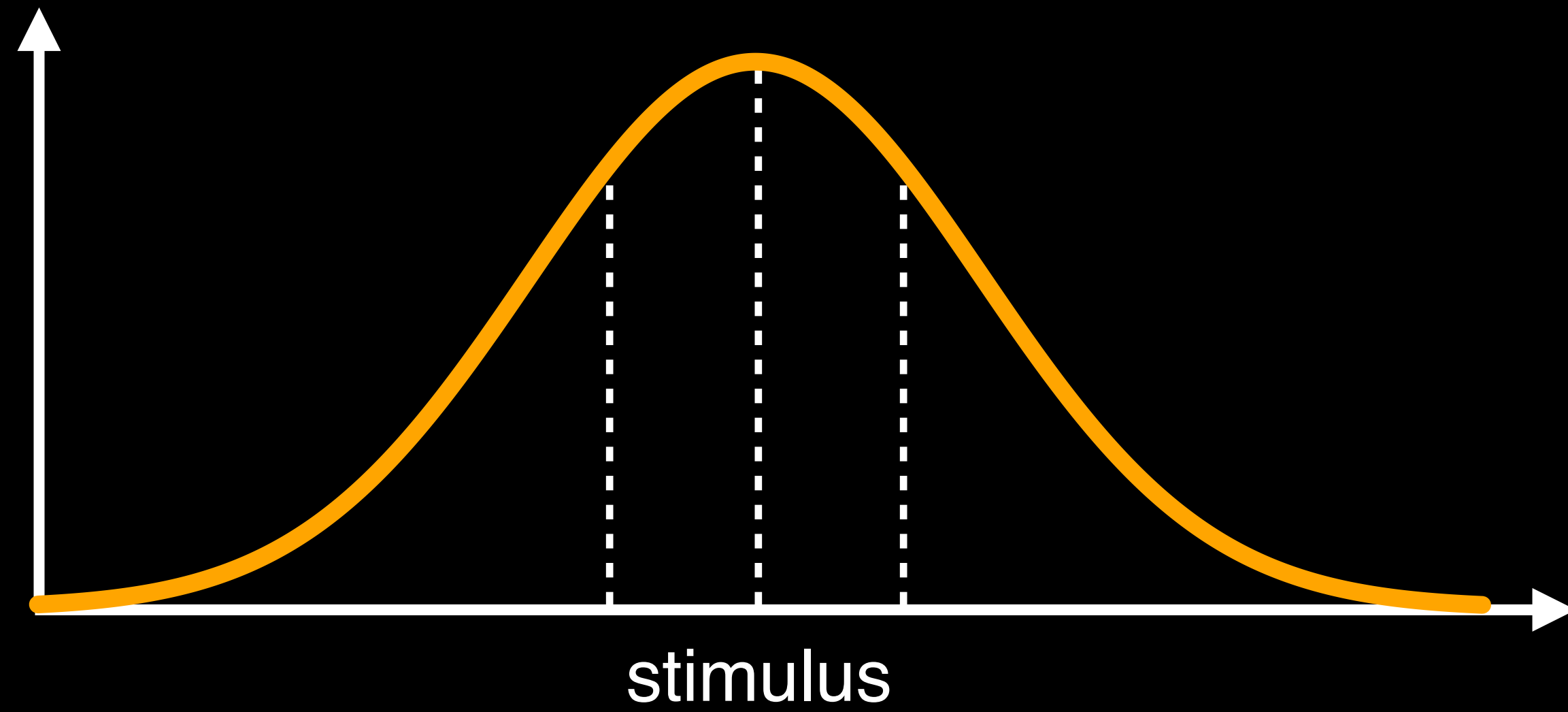
stimulus

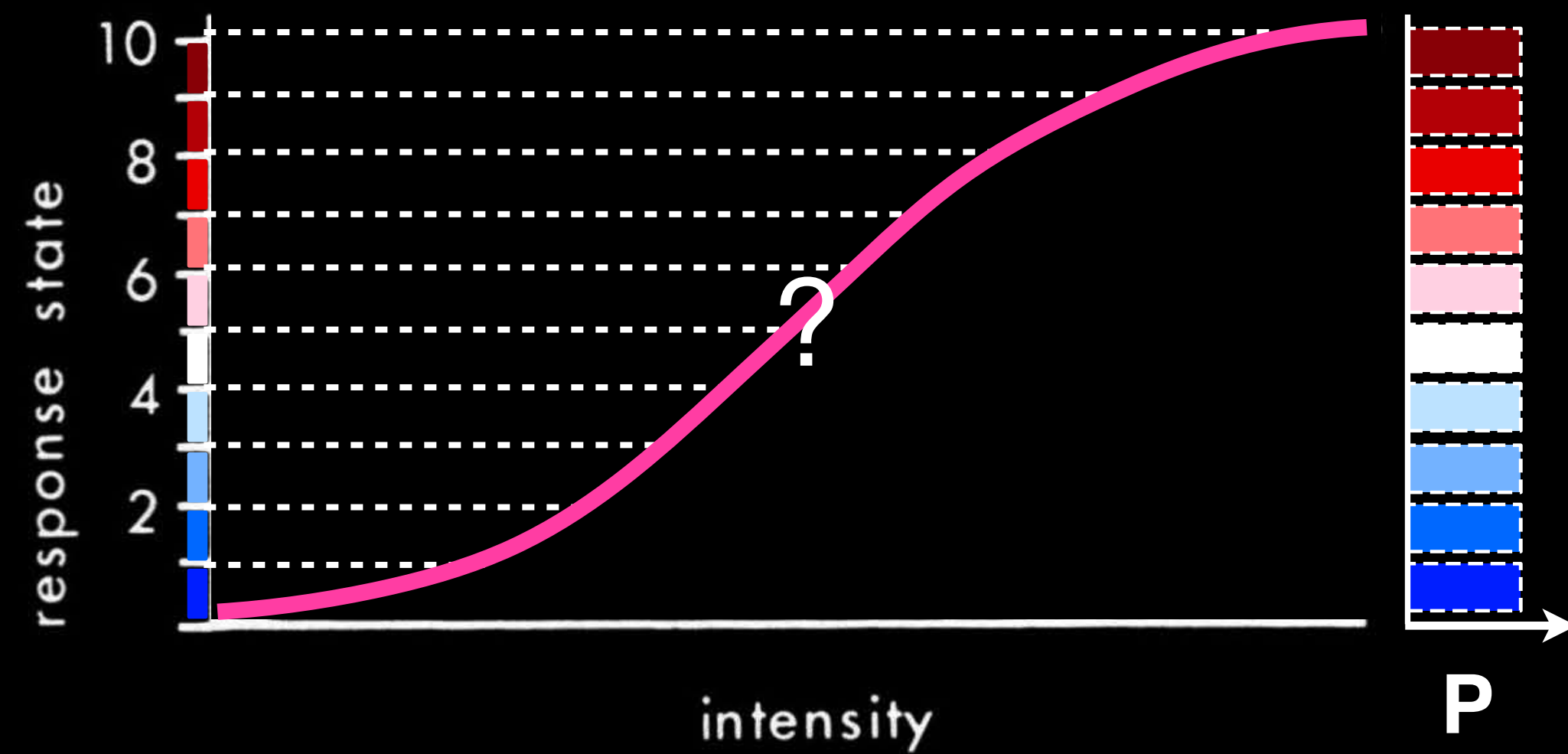
tuning curve

A B A A A B

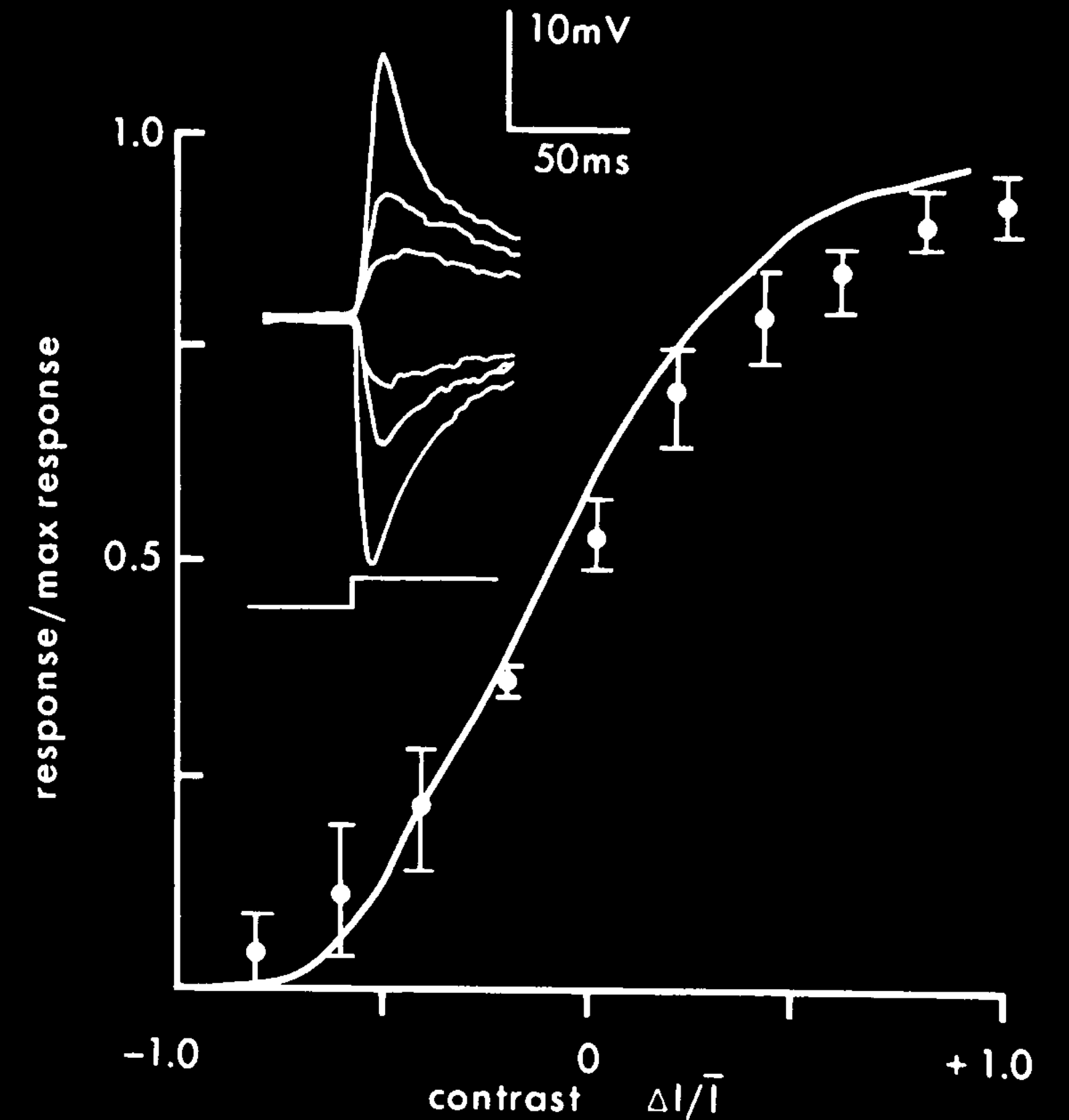


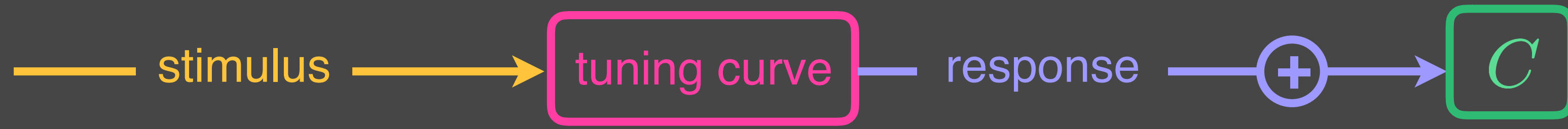




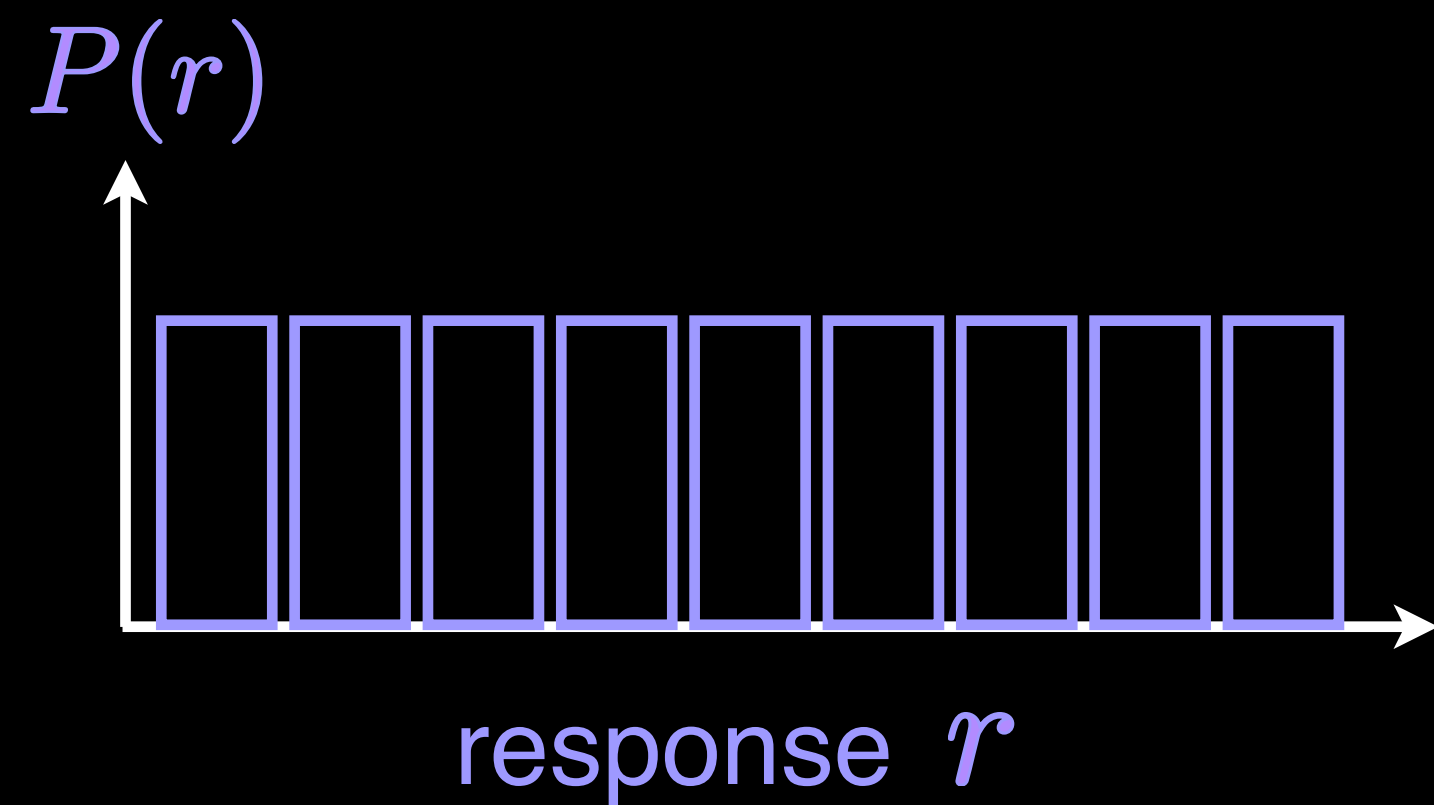


“histogram equalization”



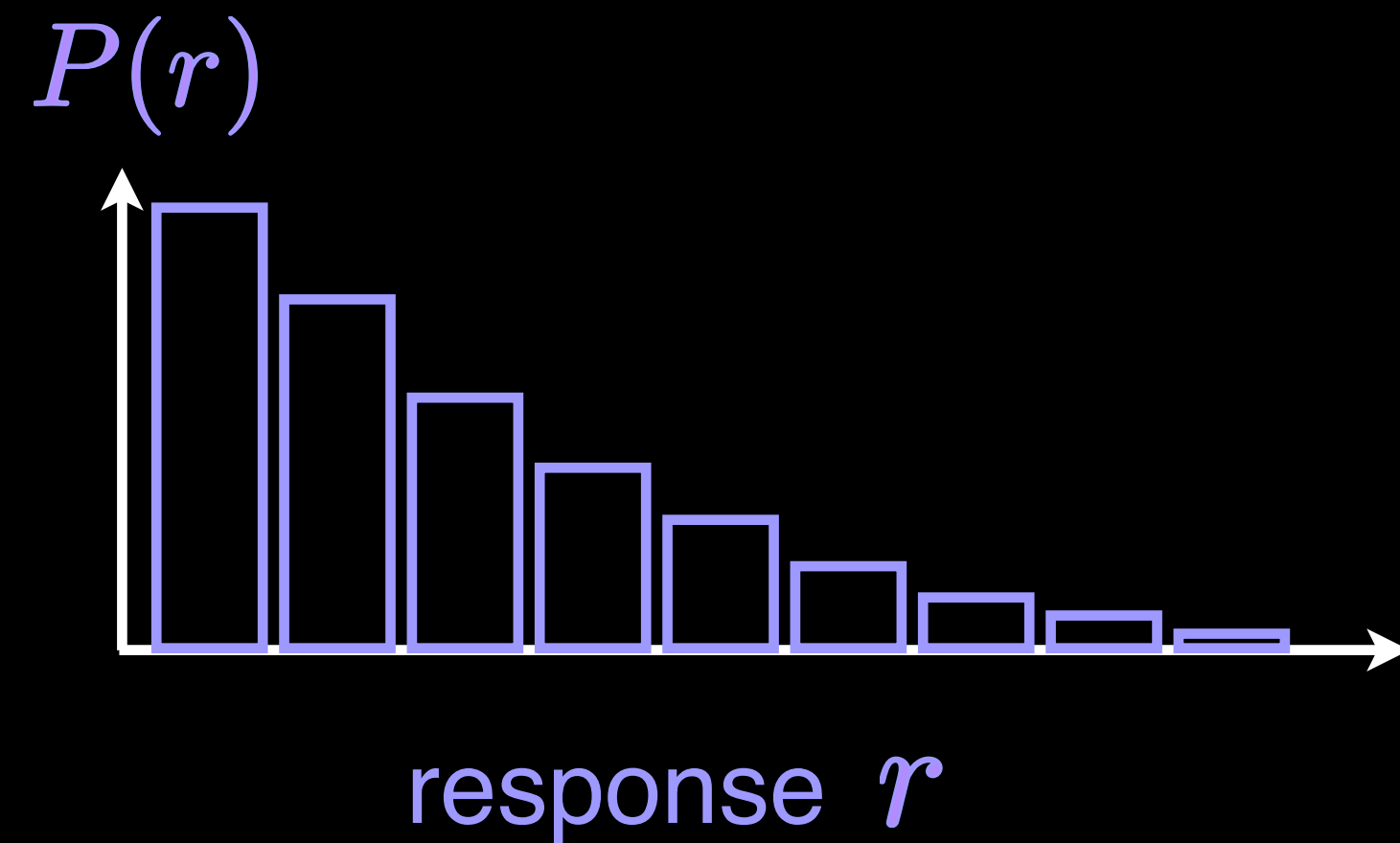


constraint on
number of responses N



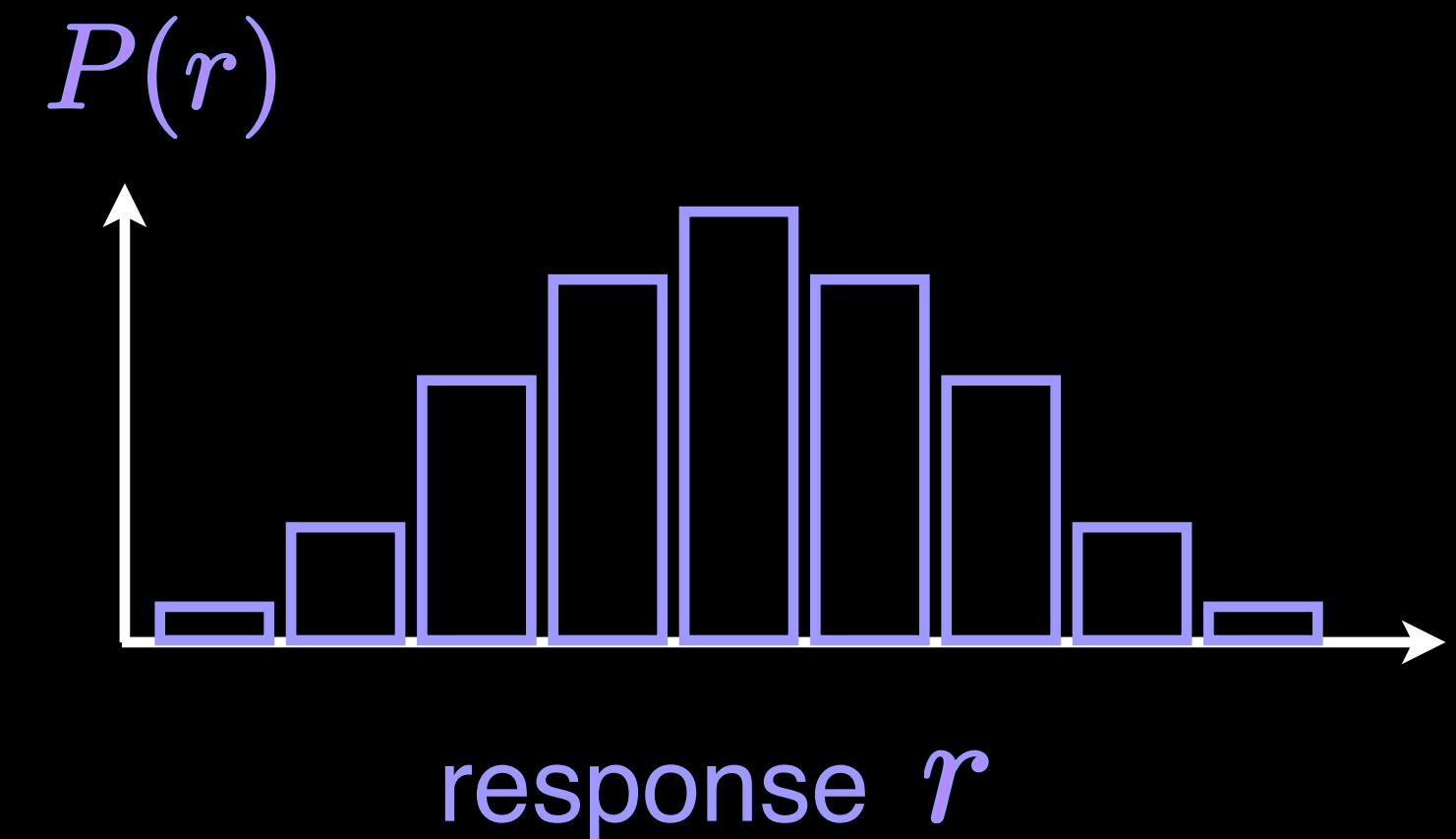
$$P(r) = 1 / N$$

constraint on
number of responses N
mean firing rate μ



$$= \frac{1}{\mu} \exp \left(-\frac{r}{\mu} \right)$$

constraint on
number of responses N
mean firing rate μ
variance in firing rate σ^2



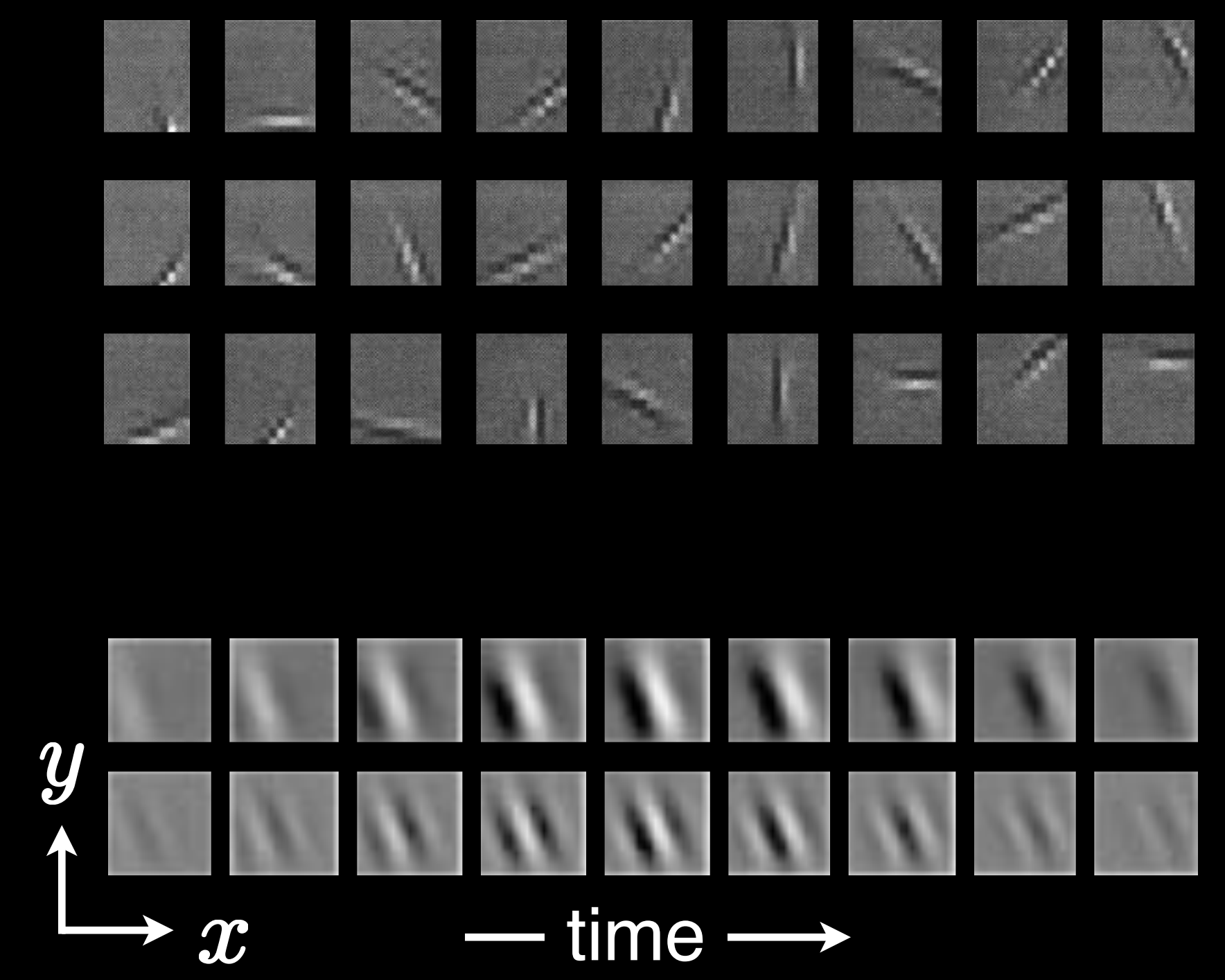
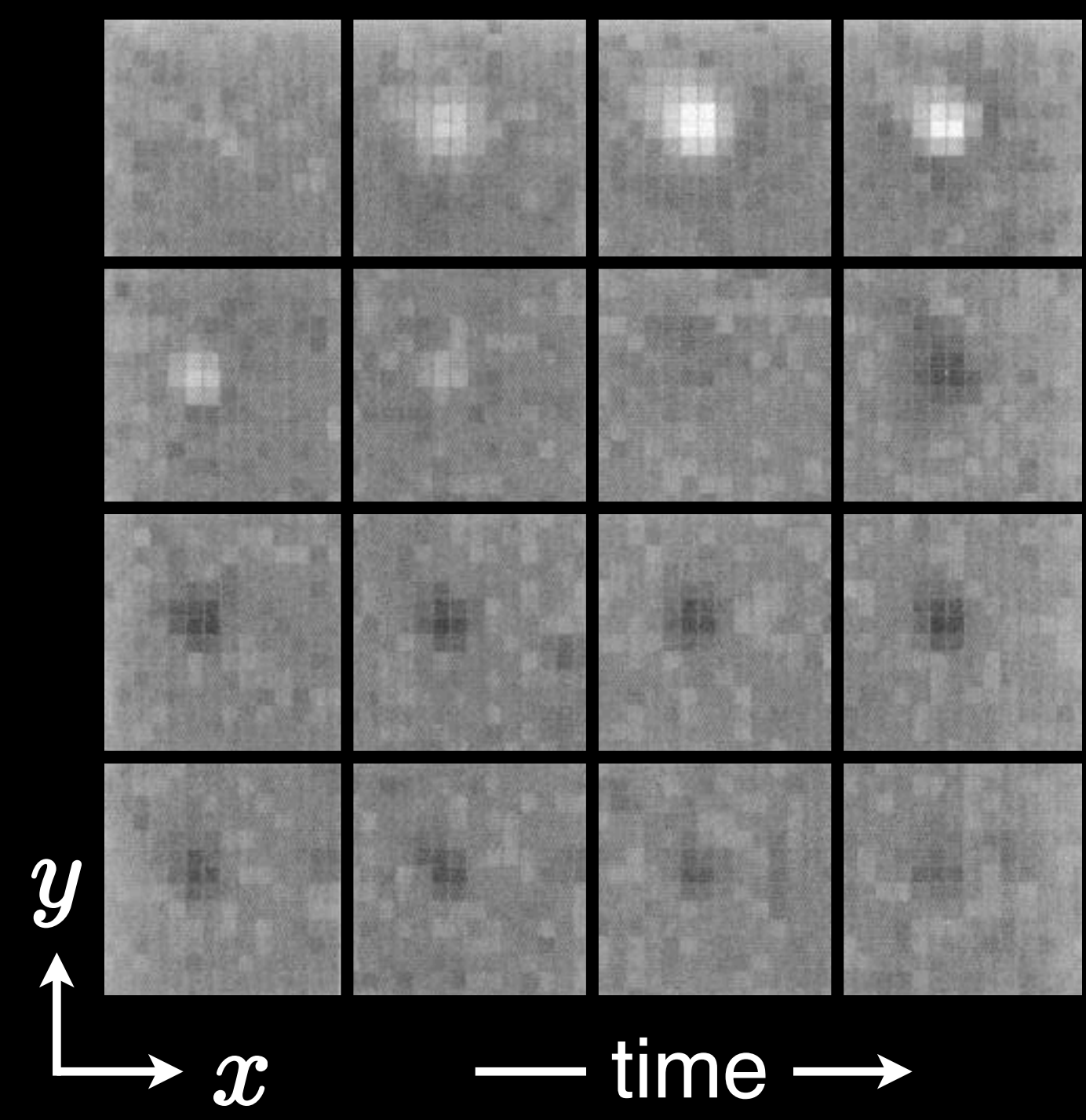
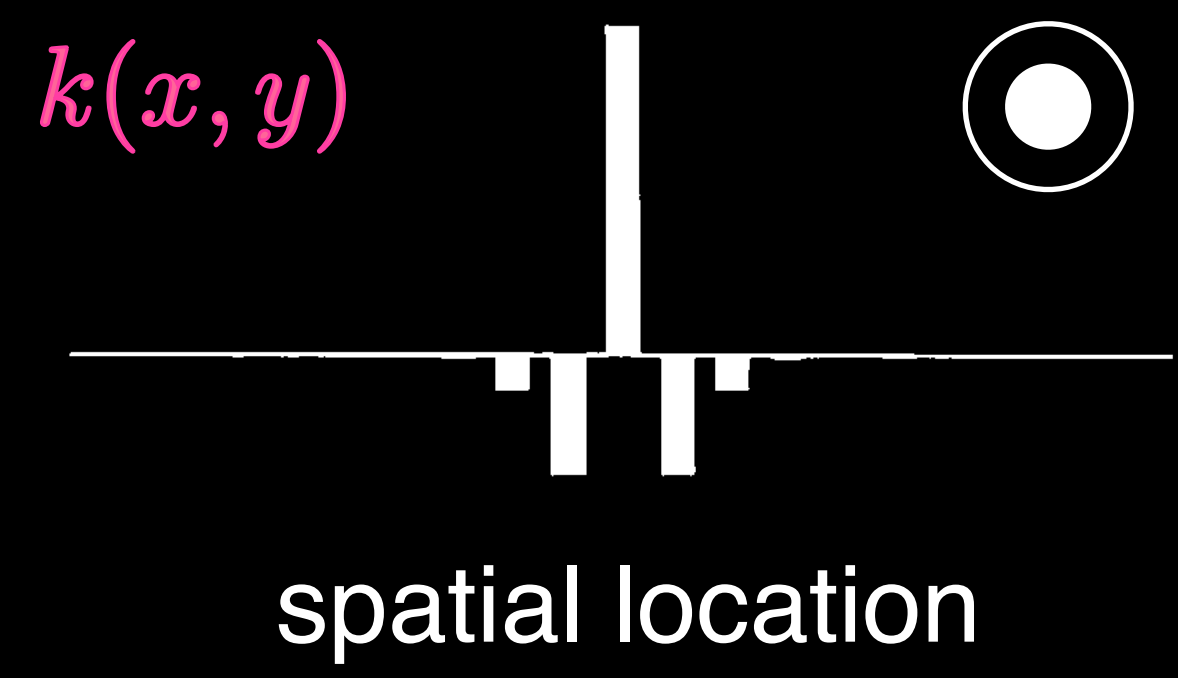
$$= \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left(-\frac{(r - \mu)^2}{2\sigma^2} \right)$$



$$\underset{\text{response}}{r(x, y)} \propto \underset{\text{linear filter}}{k(x, y)} \otimes \underset{\text{stimulus}}{s(x, y)}$$



$$r(x, y) \propto k(x, y) \circledast s(x, y)$$



Srinivasan, Laughlin, & Dubs 1982
 Atick & Redlich 1990

Dan, Atick, Reid 1996

Bell & Sejnowski, 1995, 1997
 Olshausen & Field, 1996
 van Hateren & Ruderman, 1998



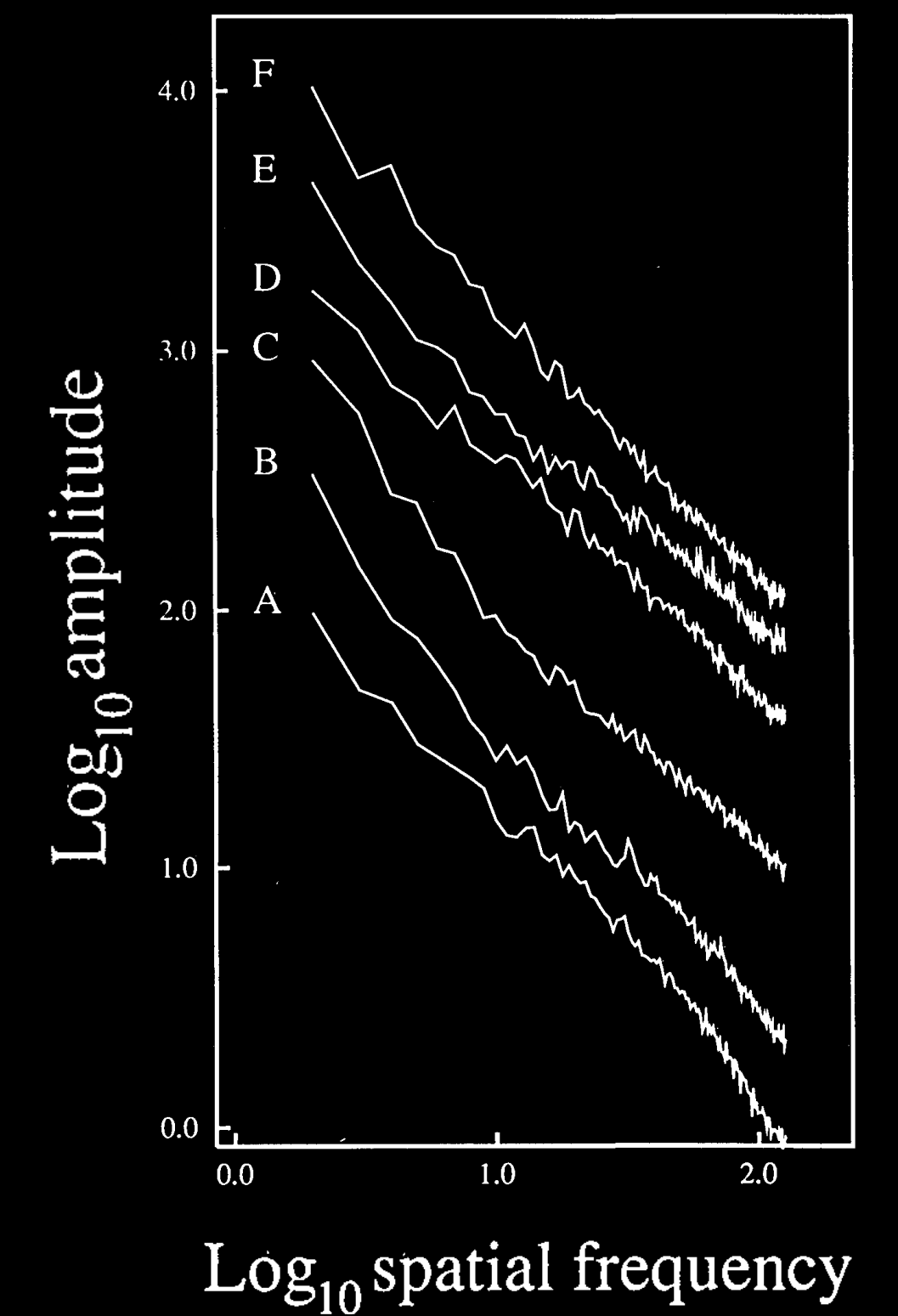
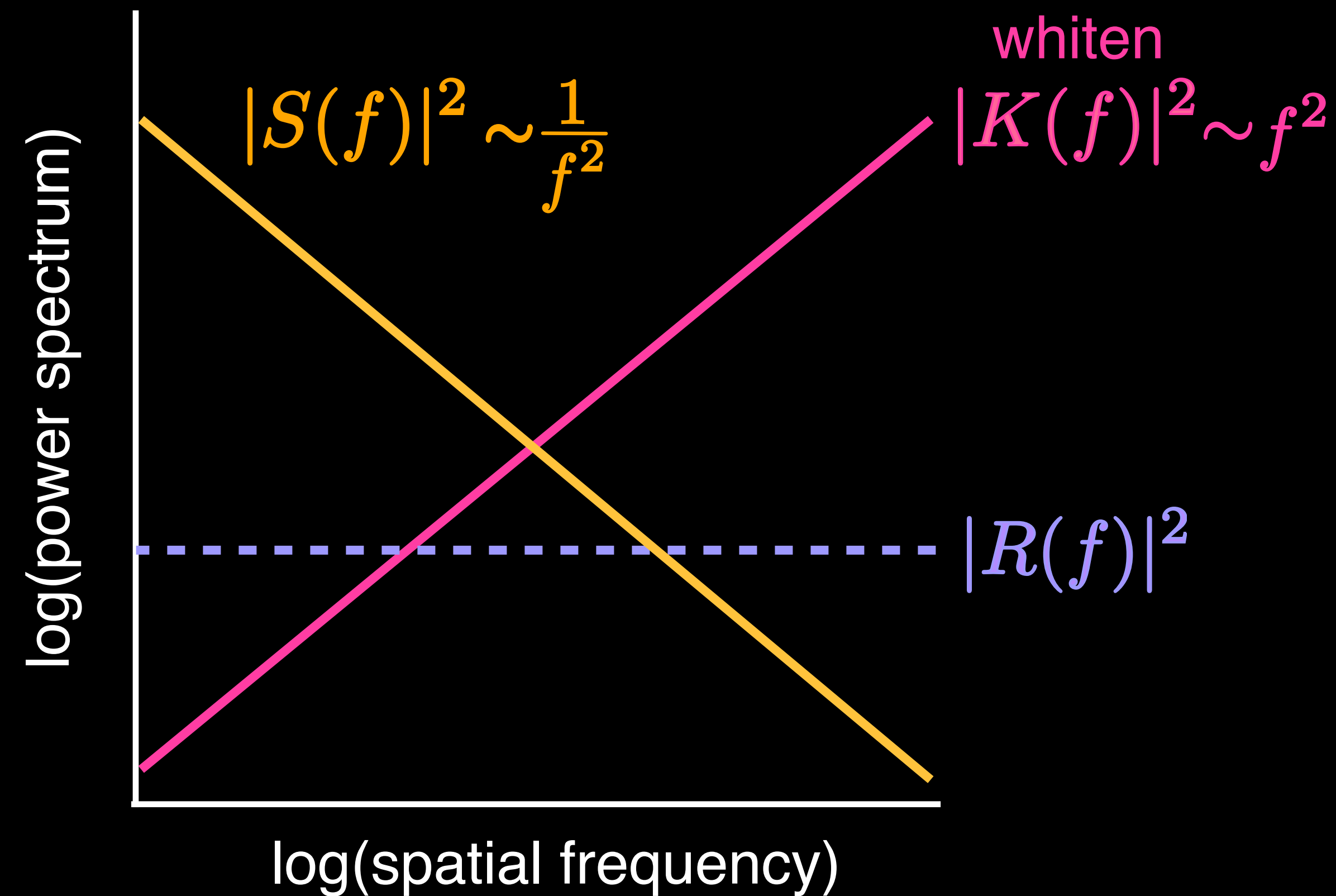
$$r(x, y) \propto k(x, y) \circledast s(x, y)$$

Fourier transform

$$|R(f)|^2 \propto |K(f)|^2 \cdot |S(f)|^2$$

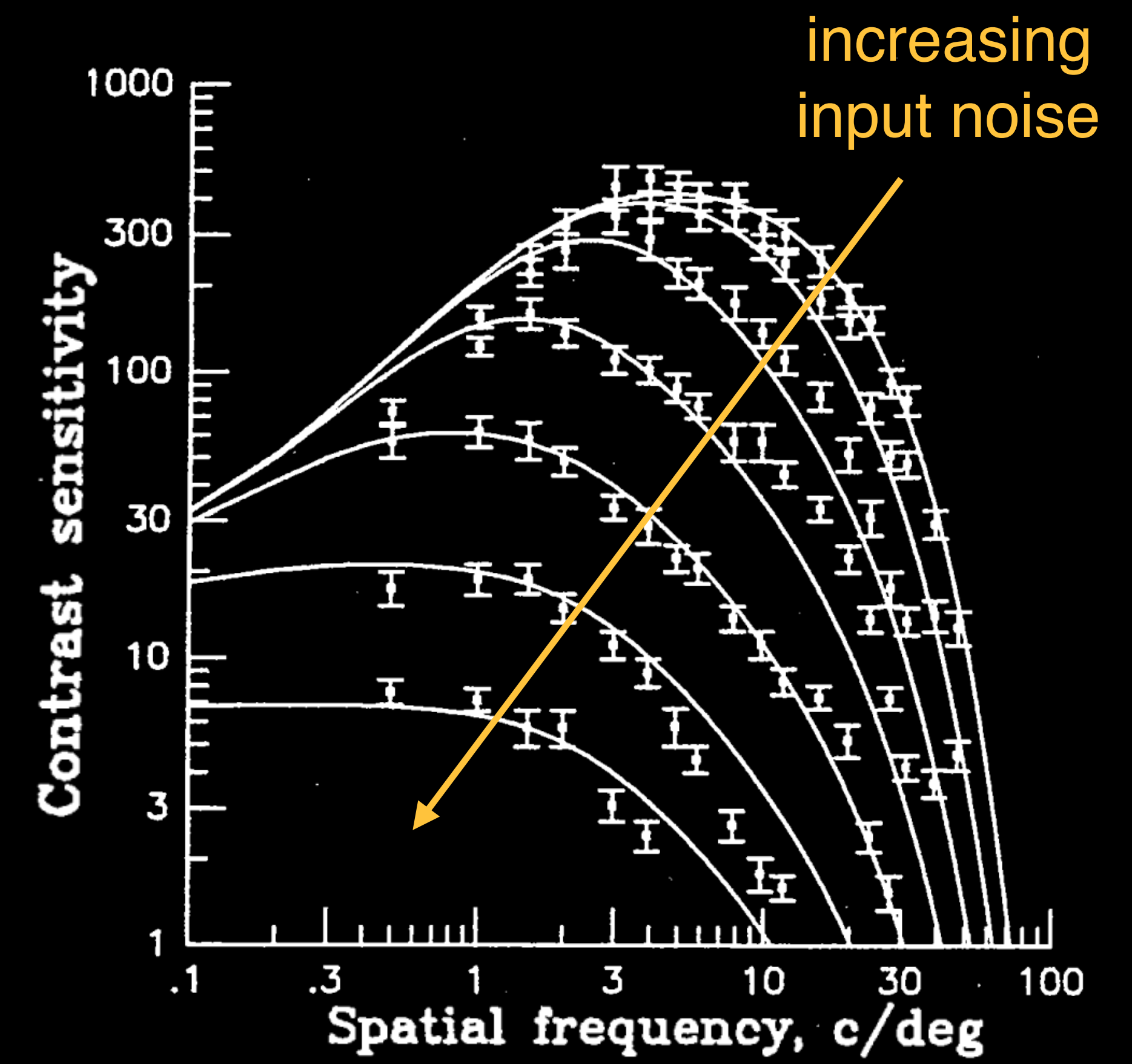
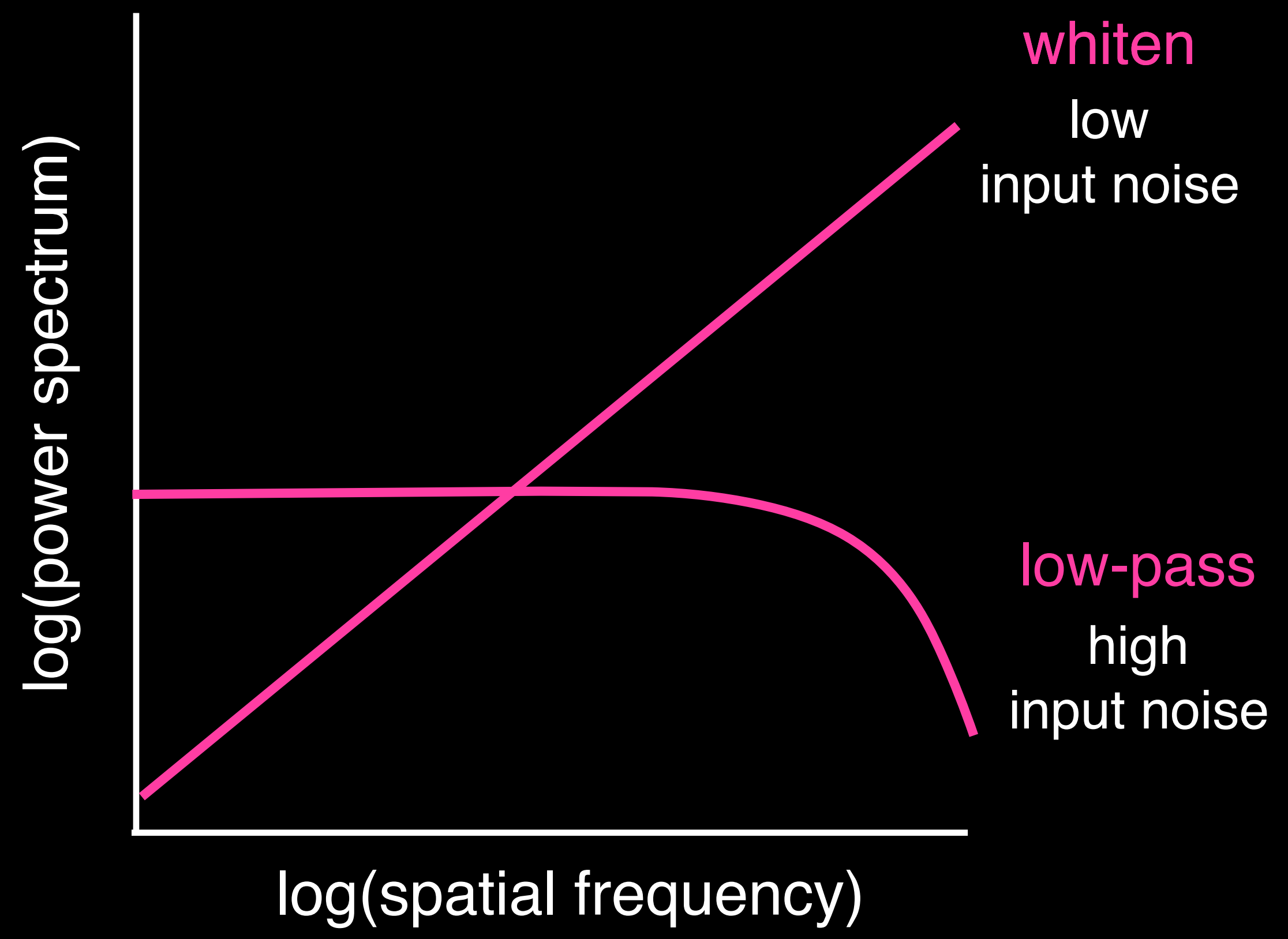


$$|R(f)|^2 \propto |K(f)|^2 \cdot |S(f)|^2$$



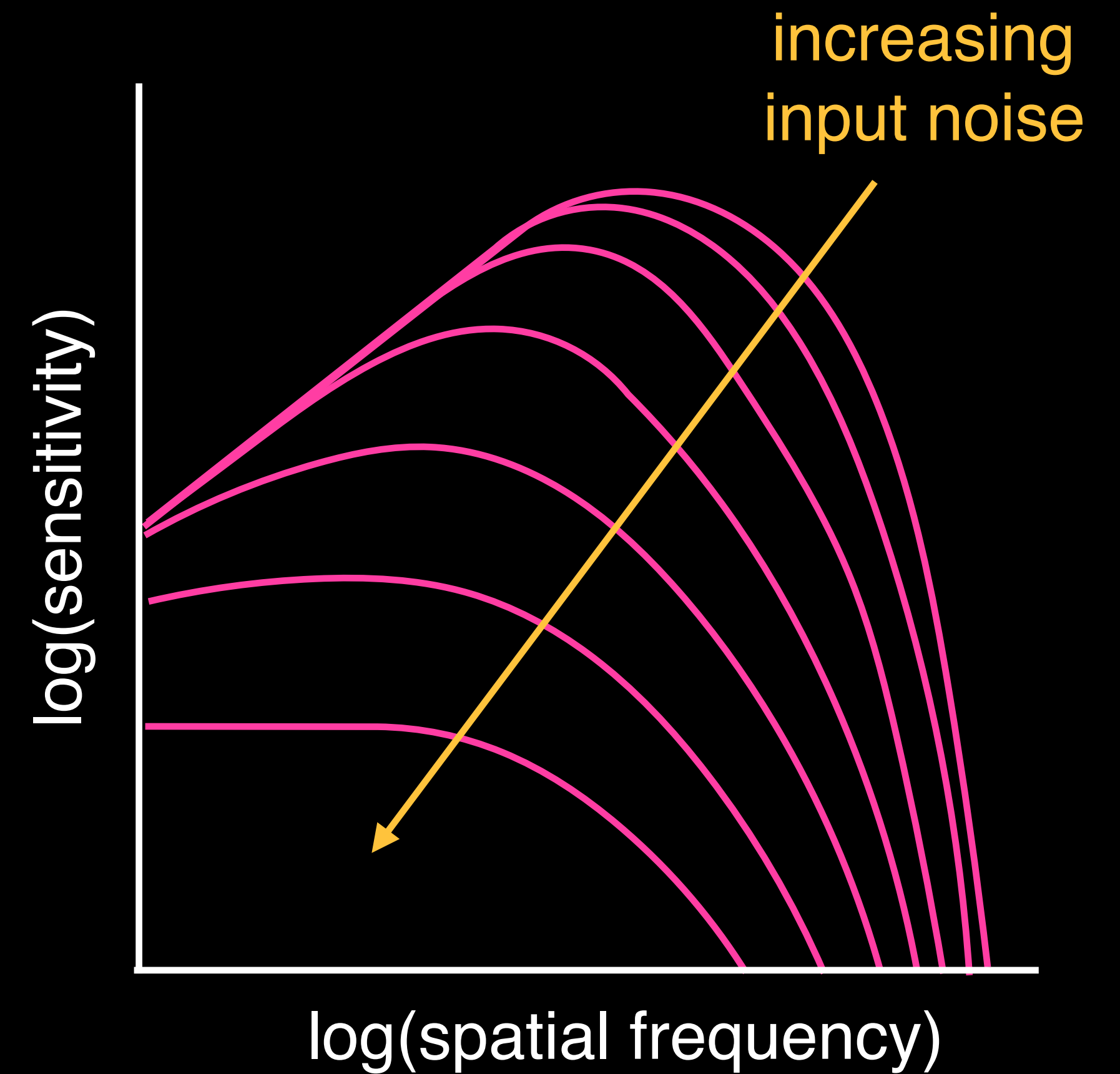
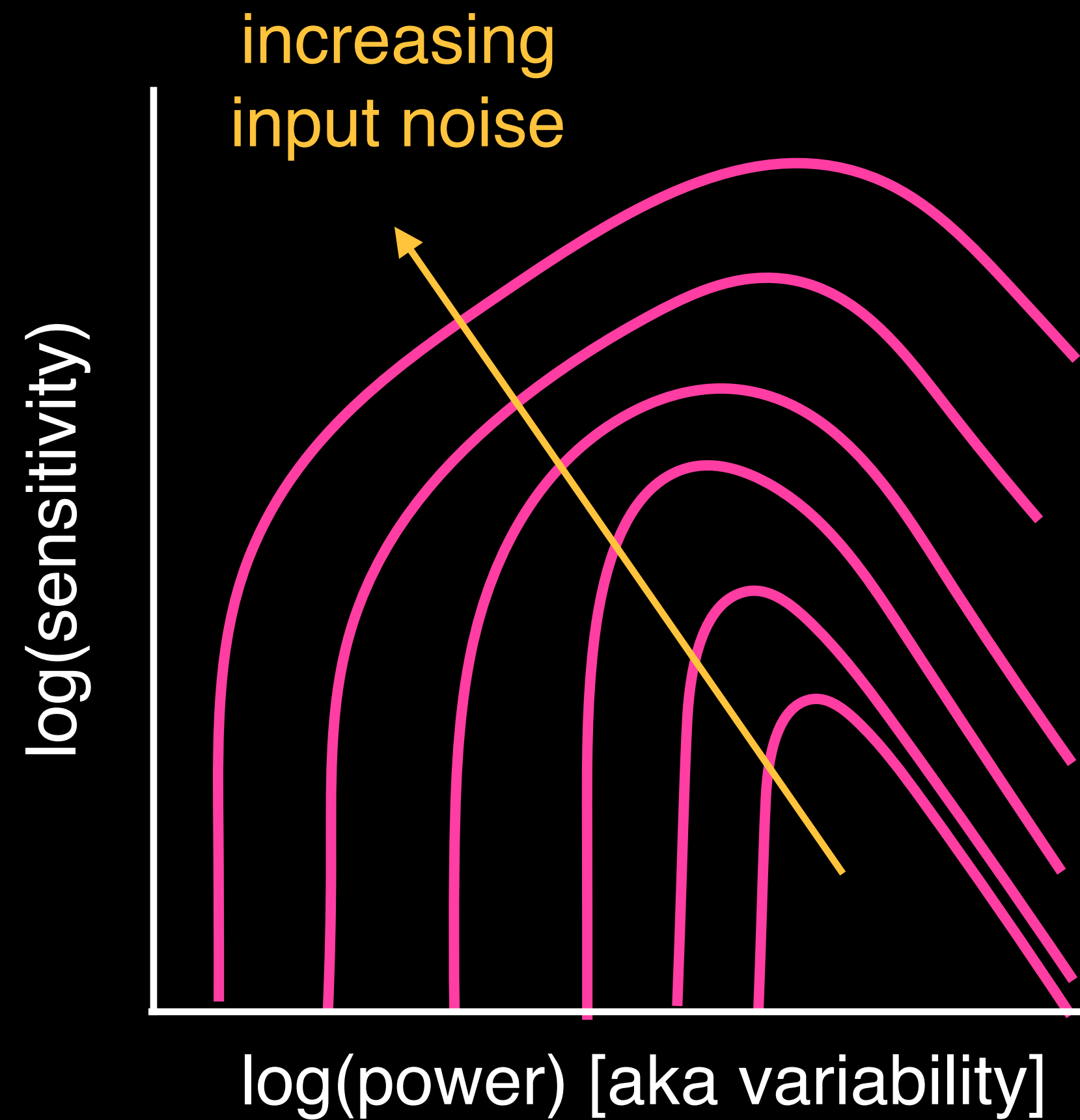


$$|R(f)|^2 \propto |K(f)|^2 \cdot |S(f)|^2$$



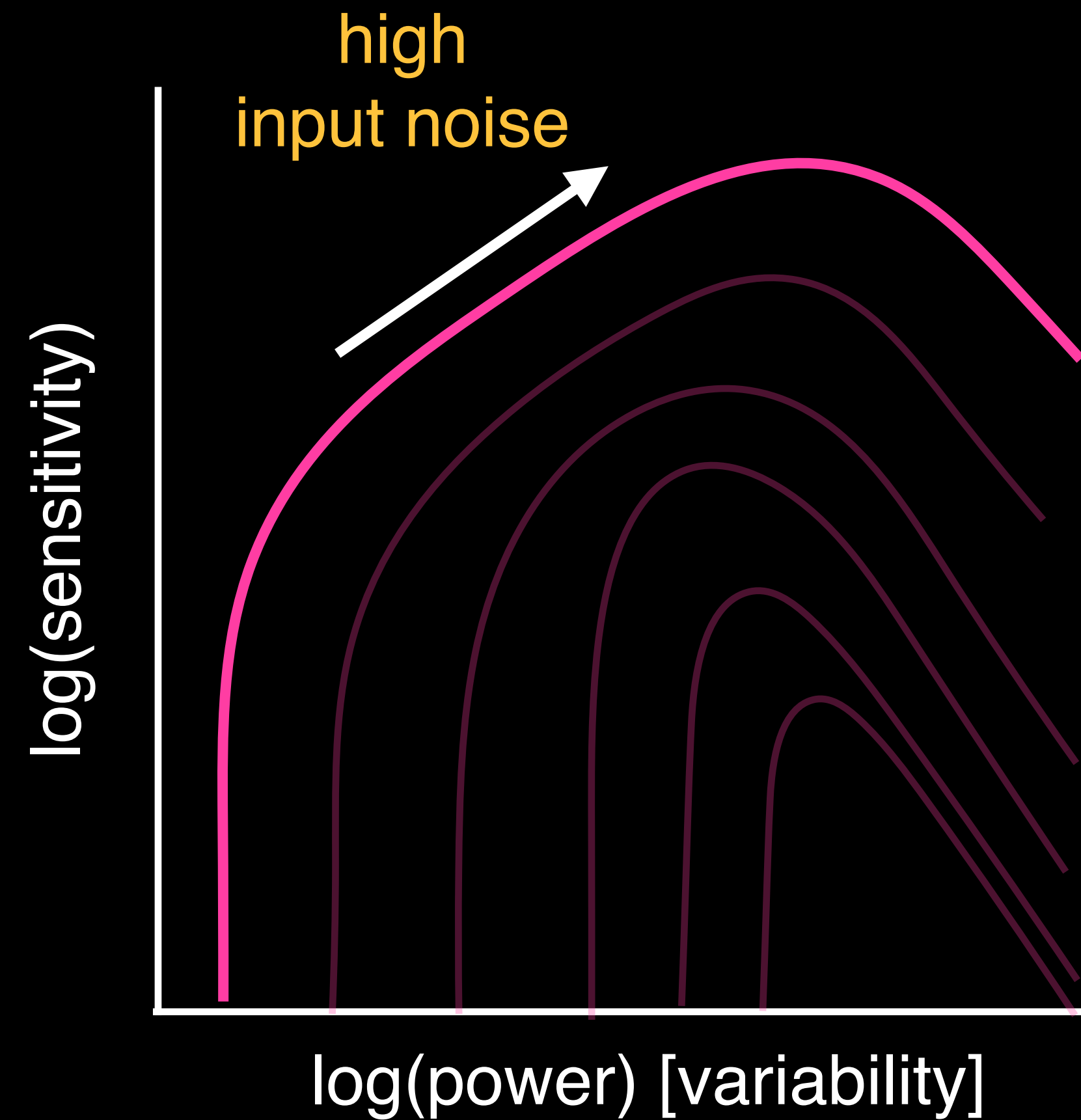


$$|R(f)|^2 \propto |K(f)|^2 \cdot |S(f)|^2$$

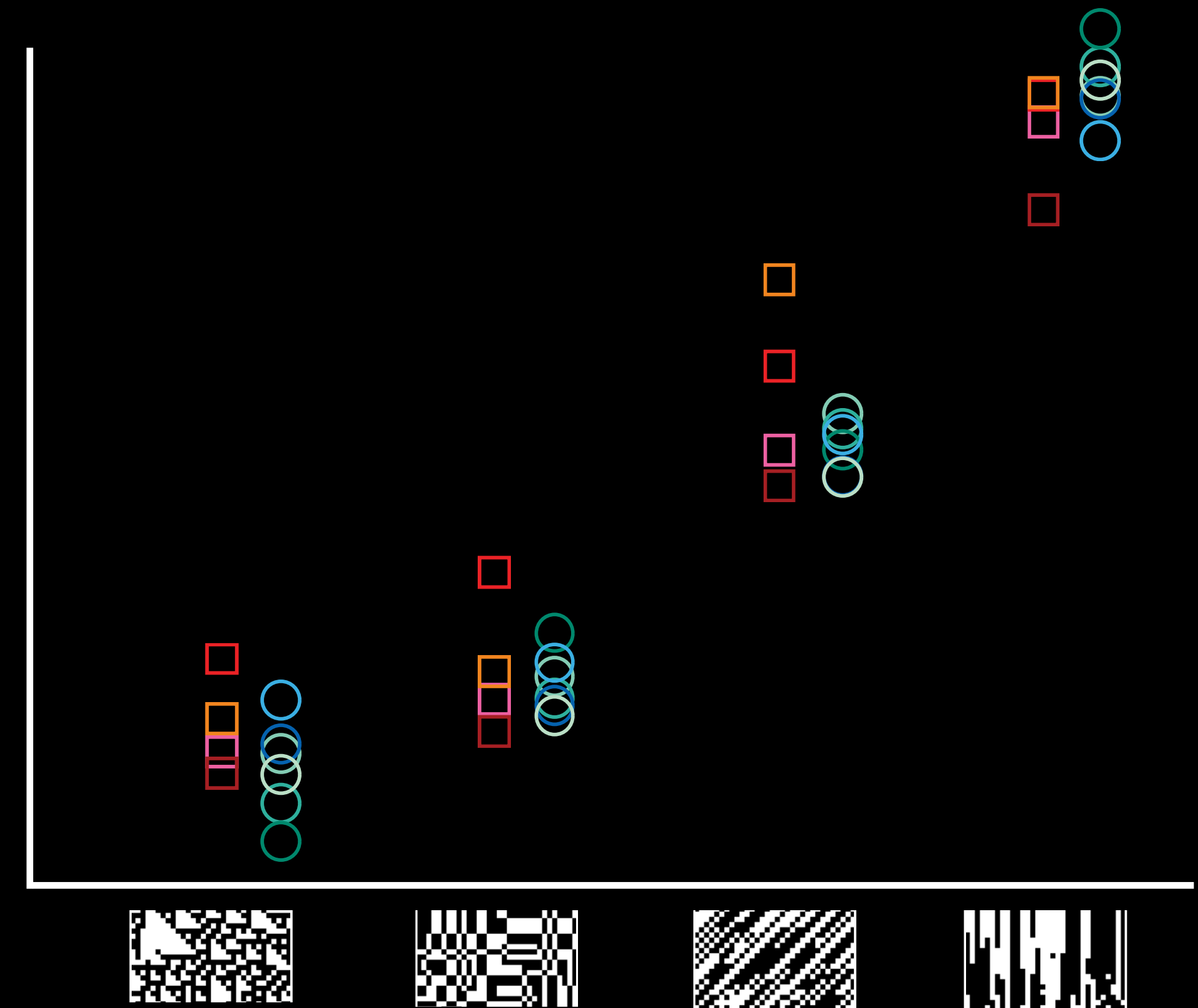




$$|R(f)|^2 \propto |K(f)|^2 \cdot |S(f)|^2$$

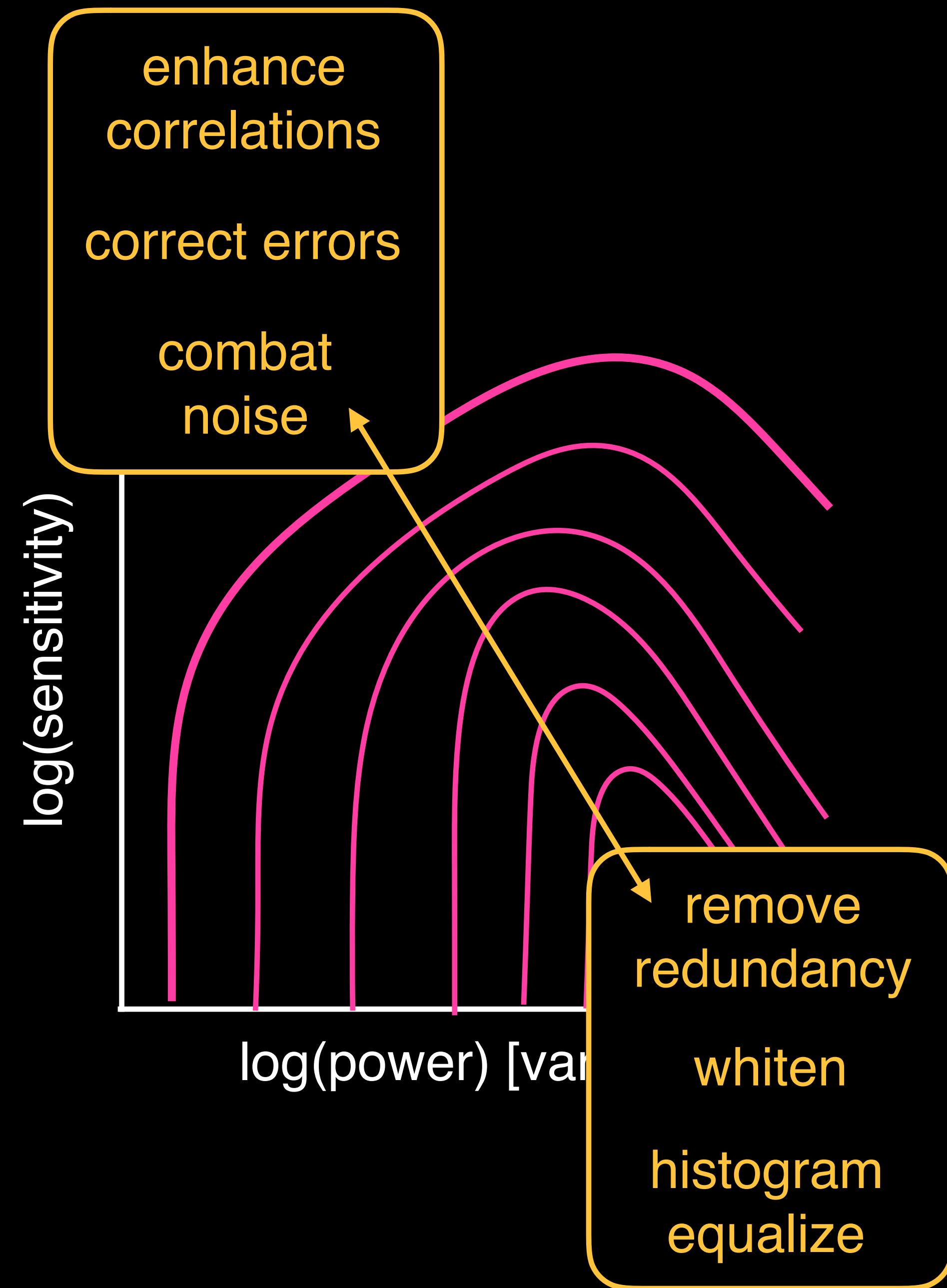
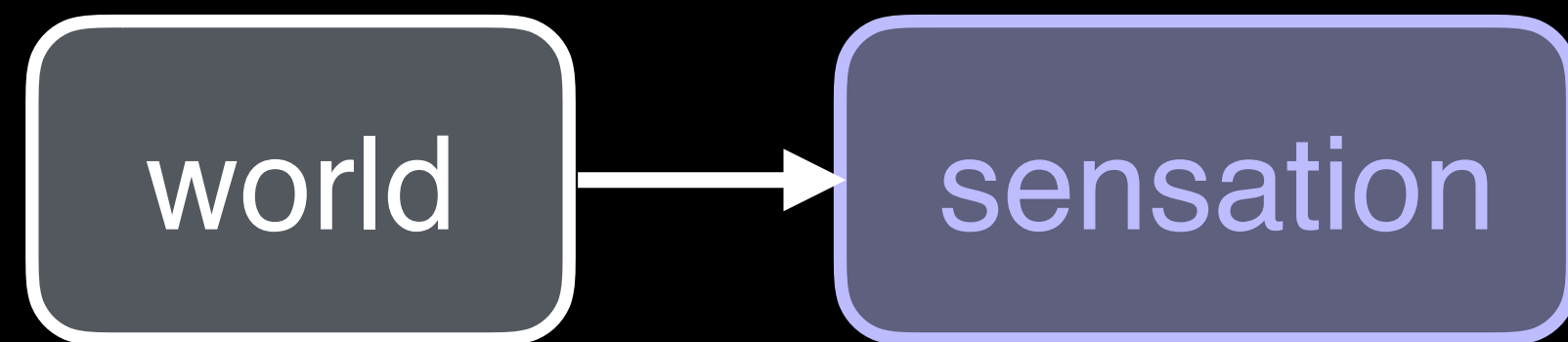


human visual sensitivity
variability in natural images



PART 1

remove redundancy
combat noise

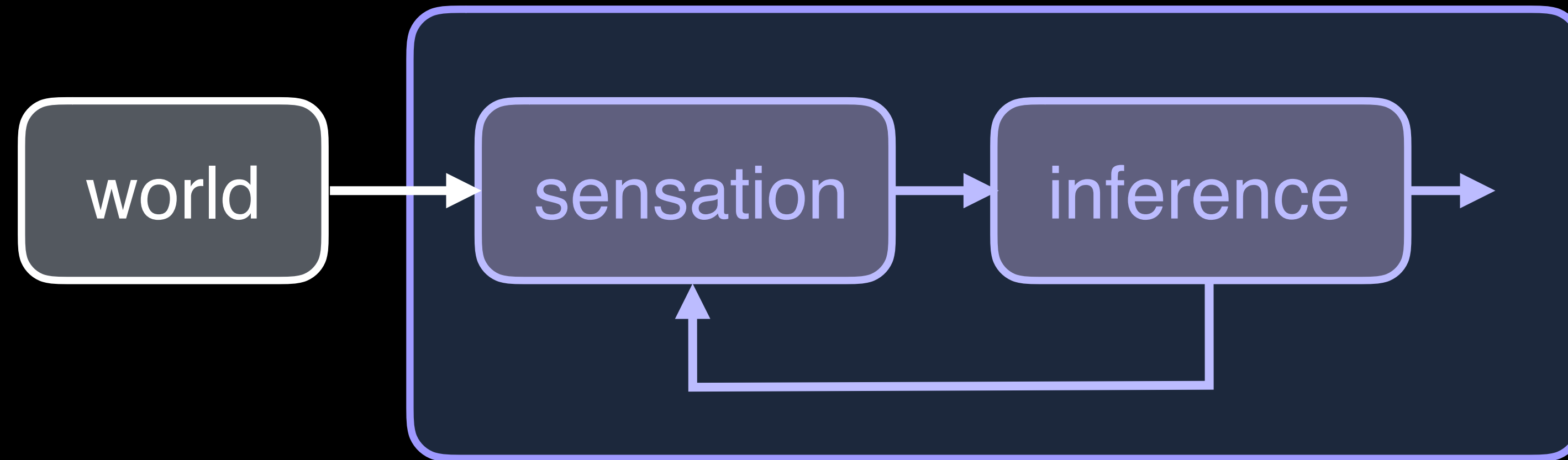


PART 1

remove redundancy
combat noise

PART 2

resolve ambiguity



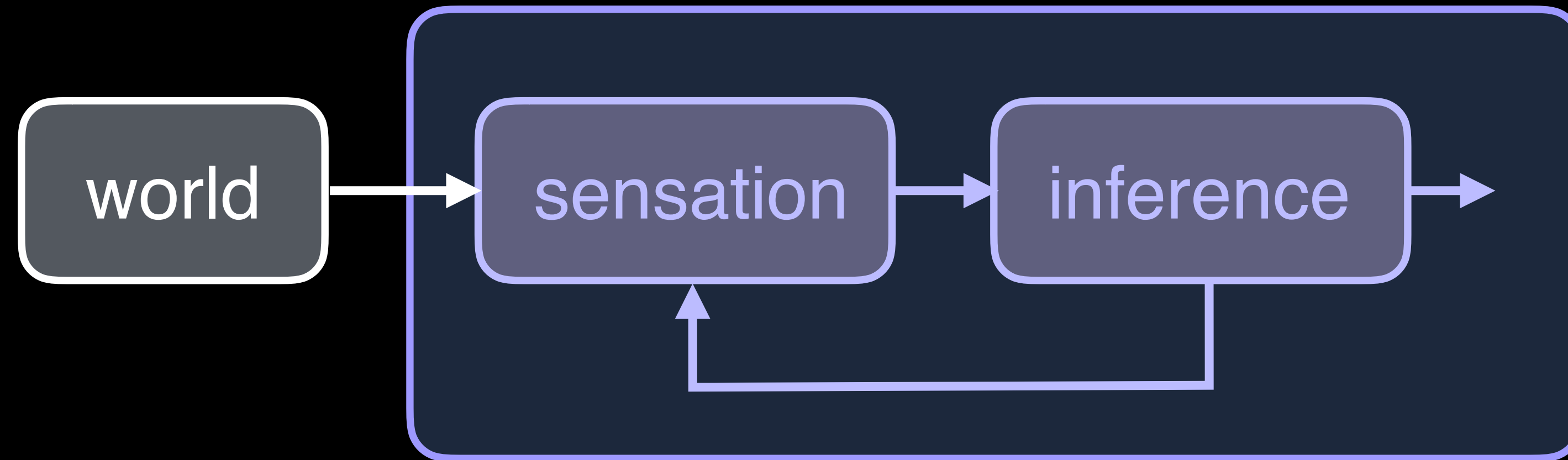
back at 1:20

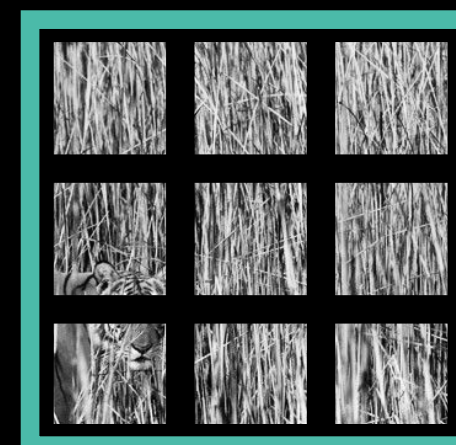
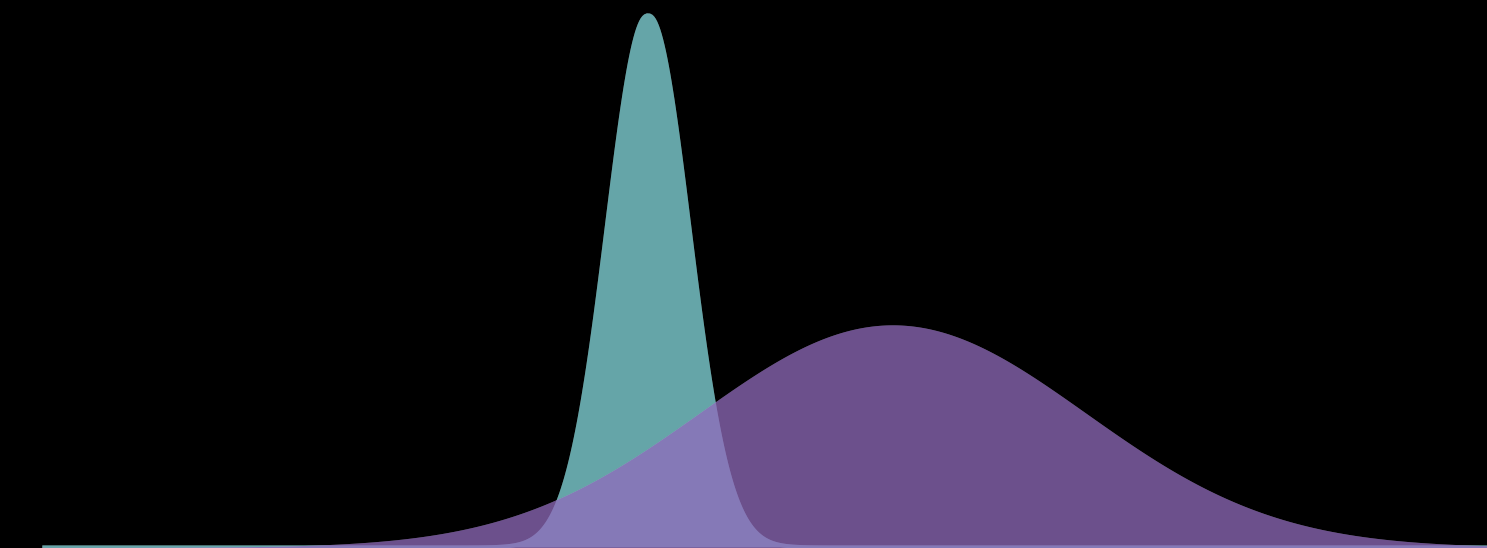
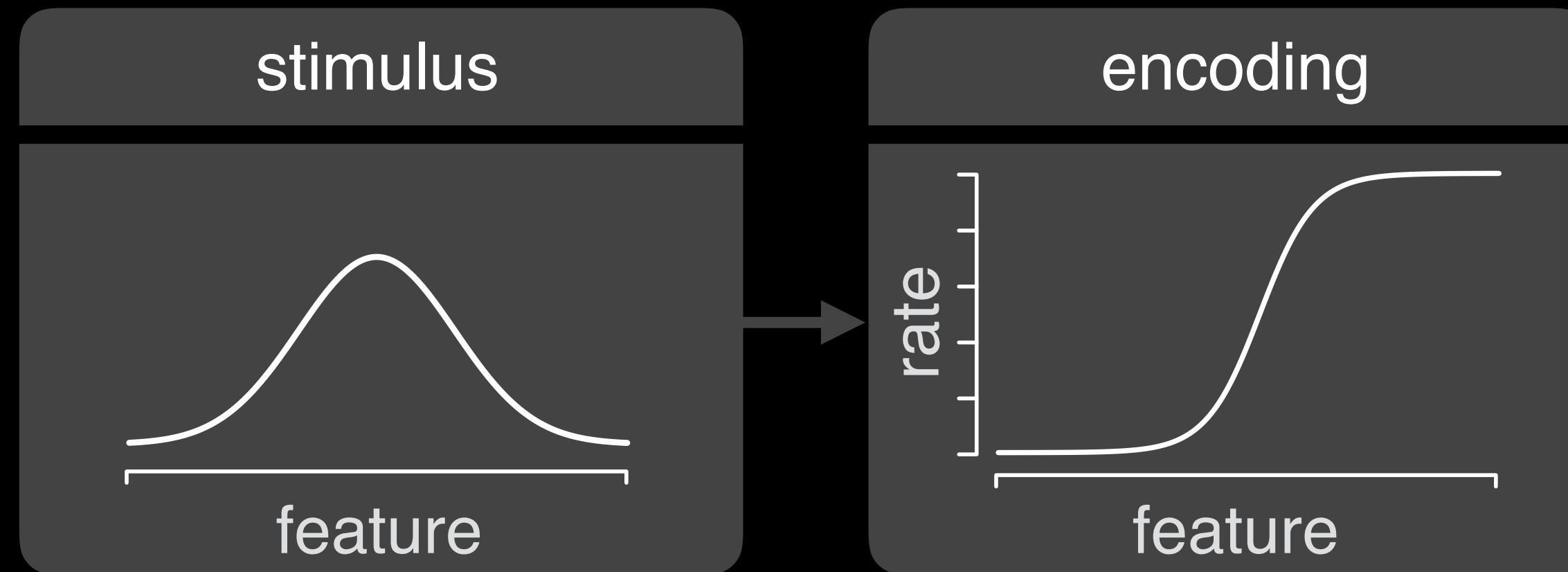
PART 1

remove redundancy
combat noise

PART 2

resolve ambiguity

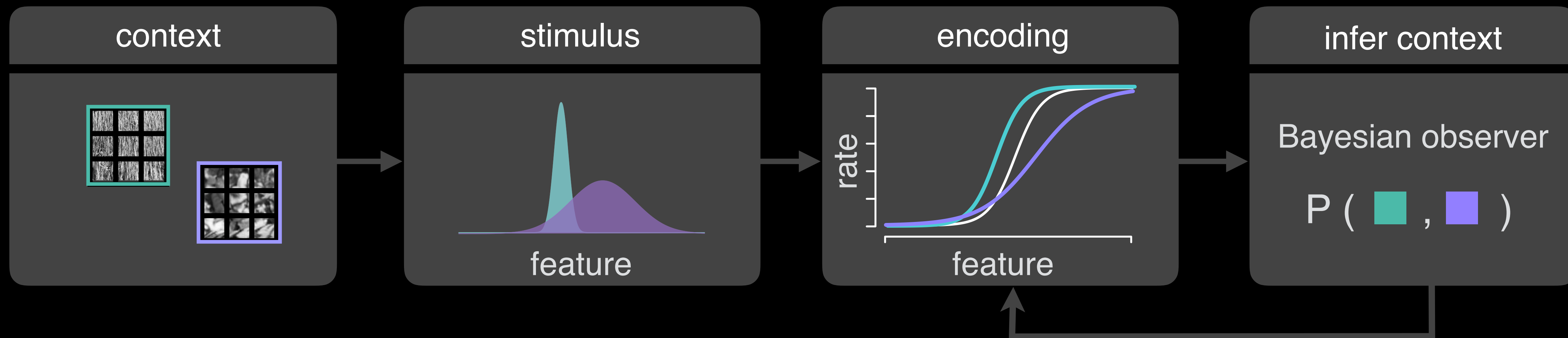


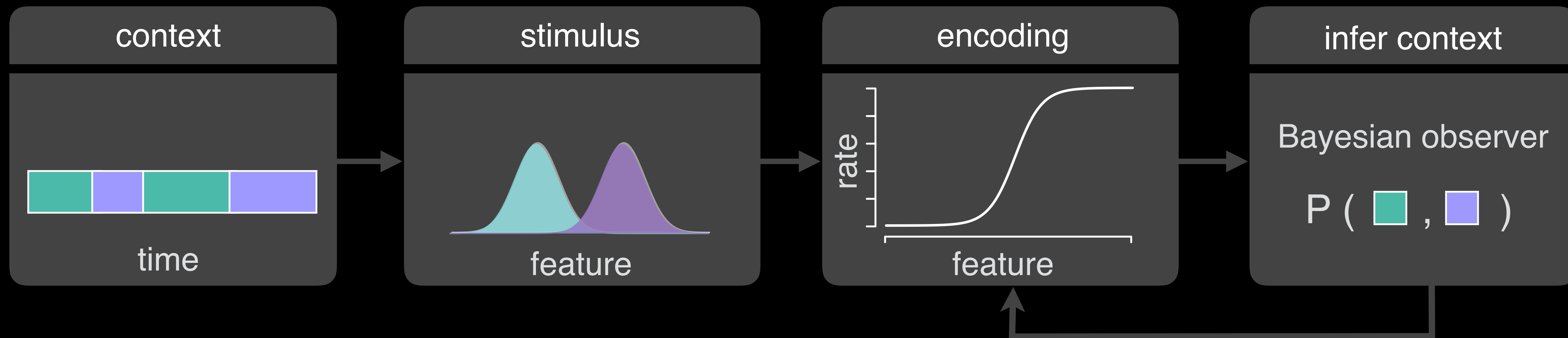


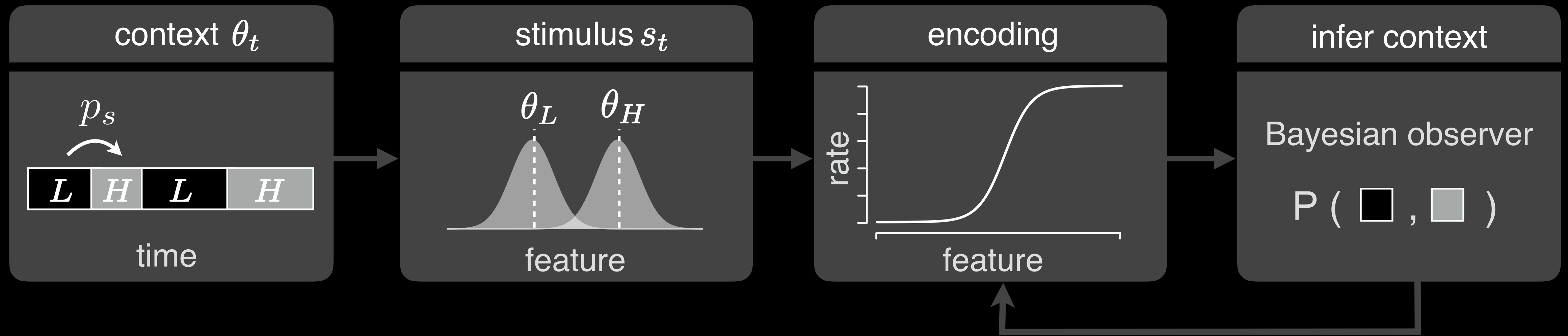
stimulus
feature

=

$$\text{stimulus feature} = \text{stimulus patch} \cdot \text{feature filter}$$







context dynamics

$$P(\theta_t | \theta_{t-1}) =$$

$$\begin{matrix} \theta_L \\ \theta_H \end{matrix} \begin{bmatrix} (1-p_s) & p_s \\ p_s & (1-p_s) \end{bmatrix}$$

$$\begin{matrix} \theta_L \\ \theta_H \end{matrix}$$

stimulus distribution

$$P(s_t | \theta_t) =$$

$$\mathcal{N}(s_t; \theta_t, \sigma^2)$$

Bayesian observer

knows $P(\theta_t | \theta_{t-1})$

$P(s_t | \theta_t)$

wants to estimate $P(\theta_t | s_t, s_{\tau < t})$

$$P(\theta_t | s_t, s_{\tau < t})$$

wants to estimate

$$P(s_t | \theta_t), P(\theta_t | \theta_{t-1})$$

knows

$$P(A, B) = P(B, A)$$

$$P(A|B)P(B) = P(B|A)P(A)$$

Bayes Rule

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

$$P(\theta_t | s_t, s_{\tau < t})$$

wants to estimate

$$P(s_t | \theta_t), P(\theta_t | \theta_{t-1})$$

knows

Bayes Rule

$$P(A | B, C) = \frac{P(B | A, C) P(A | C)}{P(B | C)}$$

$$P(\theta_t | s_t, s_{\tau < t}) = \frac{P(s_t | \theta_t, s_{\tau < t}) P(\theta_t | s_{\tau < t})}{P(s_t | s_{\tau < t})}$$

$$\begin{aligned} A &= \theta_t \\ B &= s_t \\ C &= s_{\tau < t} \end{aligned}$$

$$P(\theta_t | s_t, s_{\tau < t}) = \frac{P(s_t | \theta_t, \cancel{s_{\tau < t}}) P(\theta_t | s_{\tau < t})}{P(s_t | s_{\tau < t})}$$

$$P(\theta_t | s_t, s_{\tau < t}) = \frac{P(s_t | \theta_t) P(\theta_t | s_{\tau < t})}{P(s_t | s_{\tau < t})}$$

$$\sum_{\theta_t} P(\theta_t | s_t, s_{\tau < t}) = 1 = \sum_{\theta_t} \frac{P(s_t | \theta_t) P(\theta_t | s_{\tau < t})}{P(s_t | s_{\tau < t})}$$

$$\begin{aligned} P(s_t | s_{\tau < t}) &= \sum_{\theta_t} P(s_t | \theta_t) P(\theta_t | s_{\tau < t}) \\ &= \Omega \end{aligned}$$

$$P(\theta_t | s_t, s_{\tau < t}) = \frac{1}{\Omega} P(s_t | \theta_t) P(\theta_t | s_{\tau < t})$$

*

$$* P(\theta_t | s_{\tau < t}) = P(\theta_t | s_{\tau < t})$$

$$P(\theta_t | s_t, s_{\tau < t}) = \frac{1}{\Omega} P(s_t | \theta_t) P(\theta_t | s_{\tau < t})$$

$$* P(\theta_t | s_{\tau < t}) = \sum_{\theta_{t-1}} P(\theta_t | \theta_{t-1}, \cancel{s_{\tau < t}}) P(\theta_{t-1} | s_{\tau < t})$$

$$P(\theta_t | s_t, s_{\tau < t}) = \frac{1}{\Omega} P(s_t | \theta_t) P(\theta_t | s_{\tau < t})$$

*

$$* P(\theta_t | s_{\tau < t}) = \sum_{\theta_{t-1}} P(\theta_t | \theta_{t-1}) \frac{P(\theta_{t-1} | s_{\tau < t})}{P(\theta_{t-1} | s_{t-1}, s_{\tau < t-1})}$$

$$P(\underbrace{\theta_t}_{\textcircled{1}} | \underbrace{s_t}_{\textcircled{2}}, \underbrace{s_{\tau < t}}_{\textcircled{4}}) = \frac{1}{\Omega} P(\underbrace{s_t}_{\textcircled{2}} | \underbrace{\theta_t}_{\textcircled{3}}) \sum_{\theta_{t-1}} P(\underbrace{\theta_t}_{\textcircled{3}} | \underbrace{\theta_{t-1}}_{\textcircled{4}}) P(\underbrace{\theta_{t-1}}_{\textcircled{4}} | \underbrace{s_{\tau < t}}_{\textcircled{4}})$$

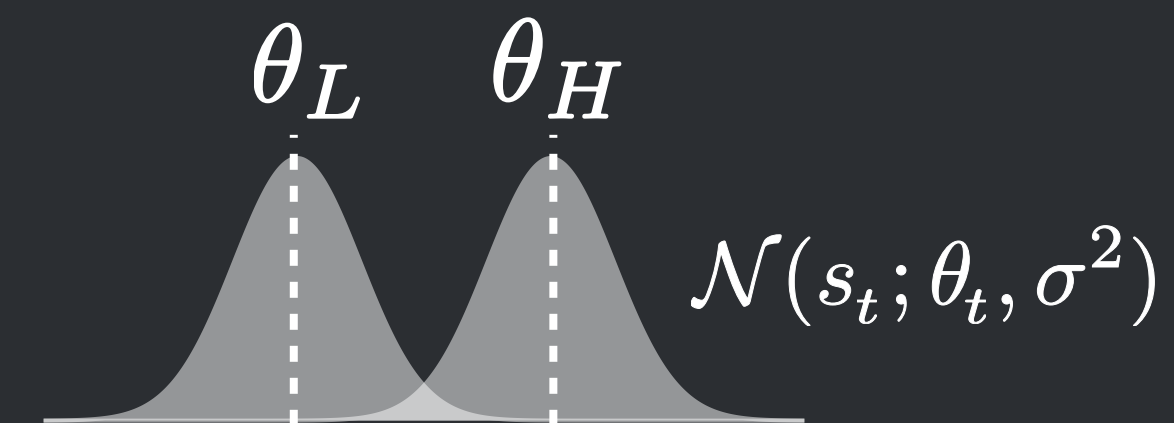
$$\textcircled{1} \quad P_t^L \equiv P(\theta_t = \theta_L | s_t, s_{\tau < t}) \quad \textcircled{3} \cdot \textcircled{4} \quad \theta_t = \theta_L \begin{bmatrix} (1-p_s) & p_s \\ p_s & (1-p_s) \end{bmatrix} \begin{matrix} \theta_L \\ \theta_H \end{matrix} \begin{bmatrix} P_{t-1}^L \\ (1 - P_{t-1}^L) \end{bmatrix}$$

$$P_t^H = (1 - P_t^L)$$

$$\textcircled{2} \quad \mathcal{N}(s_t; \theta_t = \theta_L, \sigma^2)$$



changing context θ_t



stimulus feature s_t

$$P(\underbrace{\theta_t}_{\textcircled{1}} | \underbrace{s_t}_{\textcircled{2}}, \underbrace{s_{\tau < t}}_{\textcircled{4}}) = \frac{1}{\Omega} P(\underbrace{s_t}_{\textcircled{2}} | \underbrace{\theta_t}_{\textcircled{3}}) \sum_{\theta_{t-1}} P(\underbrace{\theta_t}_{\textcircled{3}} | \underbrace{\theta_{t-1}}_{\textcircled{4}}) P(\underbrace{\theta_{t-1}}_{\textcircled{4}} | \underbrace{s_{\tau < t}}_{\textcircled{4}})$$

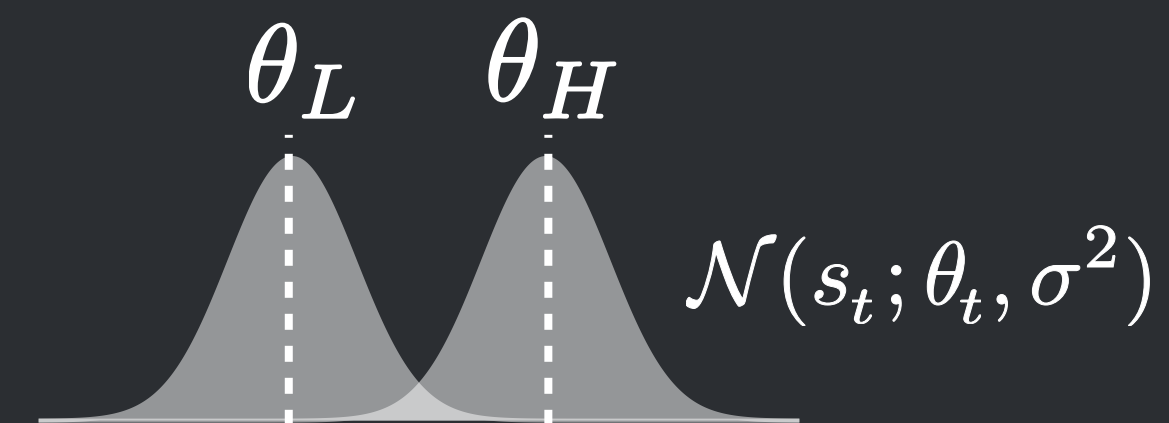
$$\textcircled{1} \quad P_t^L \equiv P(\theta_t = \theta_L | s_t, s_{\tau < t}) \quad \textcircled{3} \cdot \textcircled{4} \quad \begin{matrix} \theta_t = \theta_L \\ \theta_H \end{matrix} \begin{bmatrix} (1-p_s) & p_s \\ p_s & (1-p_s) \end{bmatrix} \begin{matrix} \theta_L \\ \theta_H \end{matrix} \begin{bmatrix} P_{t-1}^L \\ (1 - P_{t-1}^L) \end{bmatrix}$$

$$P_t^H = (1 - P_t^L)$$

$$\textcircled{2} \quad \mathcal{N}(s_t; \theta_t = \theta_L, \sigma^2) = \left[(1-p_s) P_{t-1}^L + p_s (1 - P_{t-1}^L) \right]$$



changing context θ_t



stimulus feature s_t

$$P(\theta_t | s_t, s_{\tau < t}) = \frac{1}{\Omega} P(s_t | \theta_t) \sum_{\theta_{t-1}} P(\theta_t | \theta_{t-1}) P(\theta_{t-1} | s_{\tau < t})$$

$$P_t^L = \frac{1}{\Omega} \mathcal{N}(s_t; \theta_t = \theta_L, \sigma^2) \left[(1 - p_s) P_{t-1}^L + p_s (1 - P_{t-1}^L) \right]$$

$$P_t^L = \frac{1}{\Omega} \mathcal{N}(s_t; \theta_t = \theta_L, \sigma^2) \left[(1 - p_s) P_{t-1}^L + p_s (1 - P_{t-1}^L) \right]$$

posterior
probability of
LOW context

likelihood that
observed stimulus
was generated
in LOW context

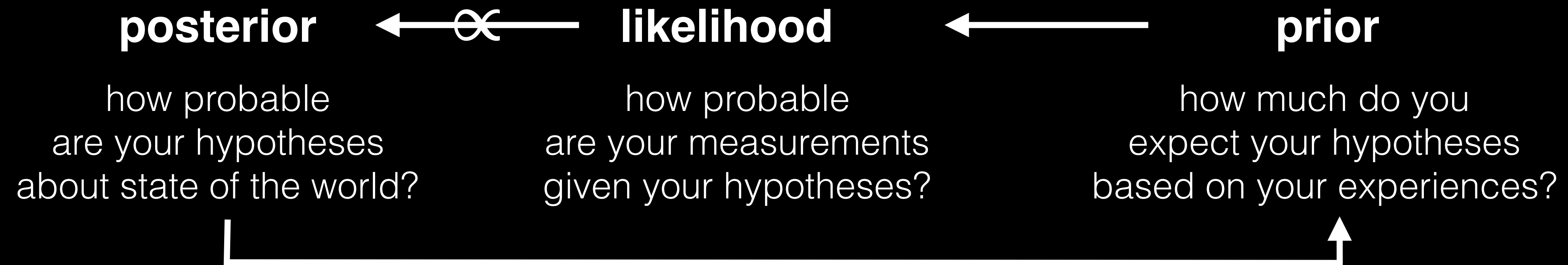
probability
that context
stayed LOW

prior
probability of
LOW context

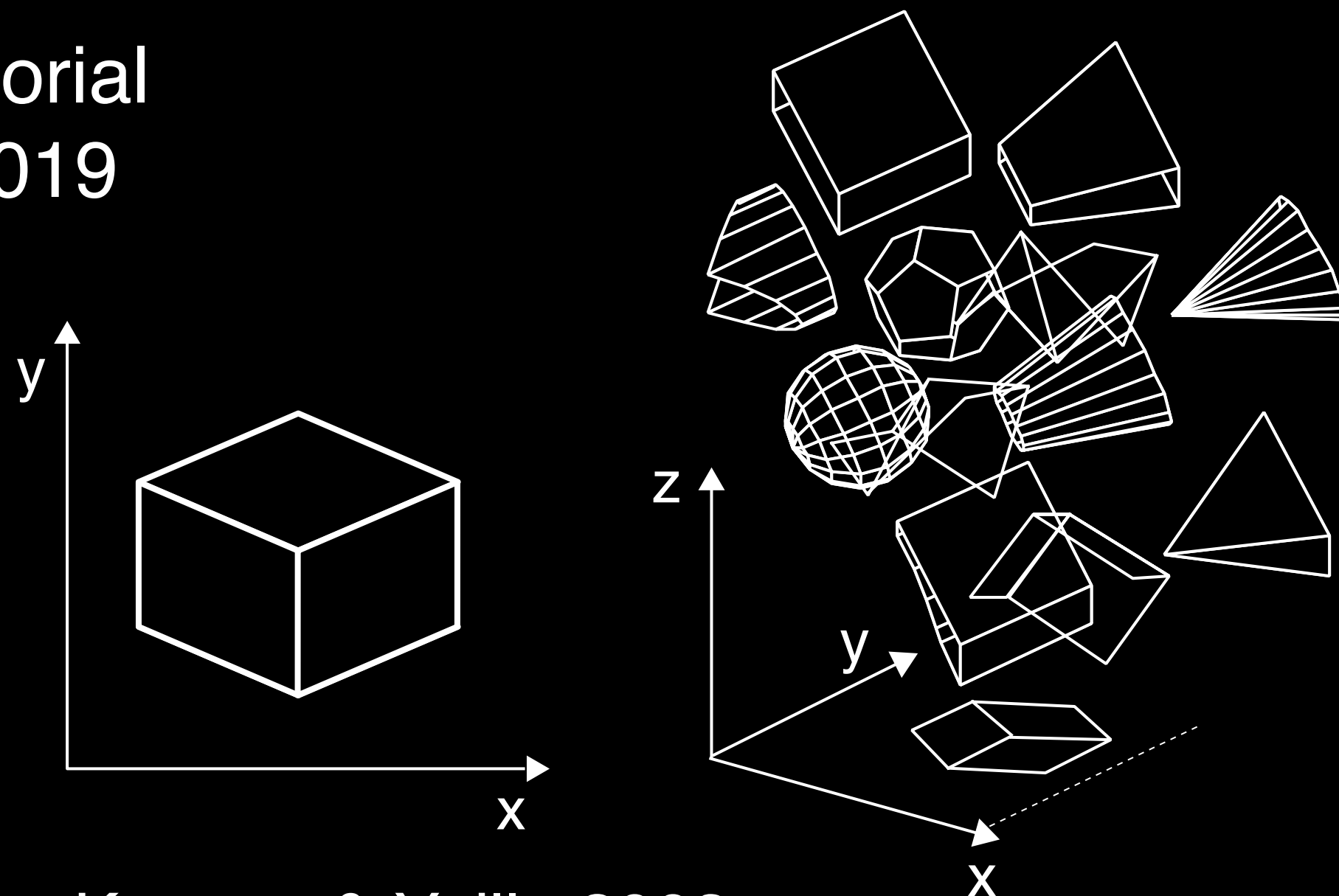
probability
that context
changed to HIGH

prior
probability of
HIGH context

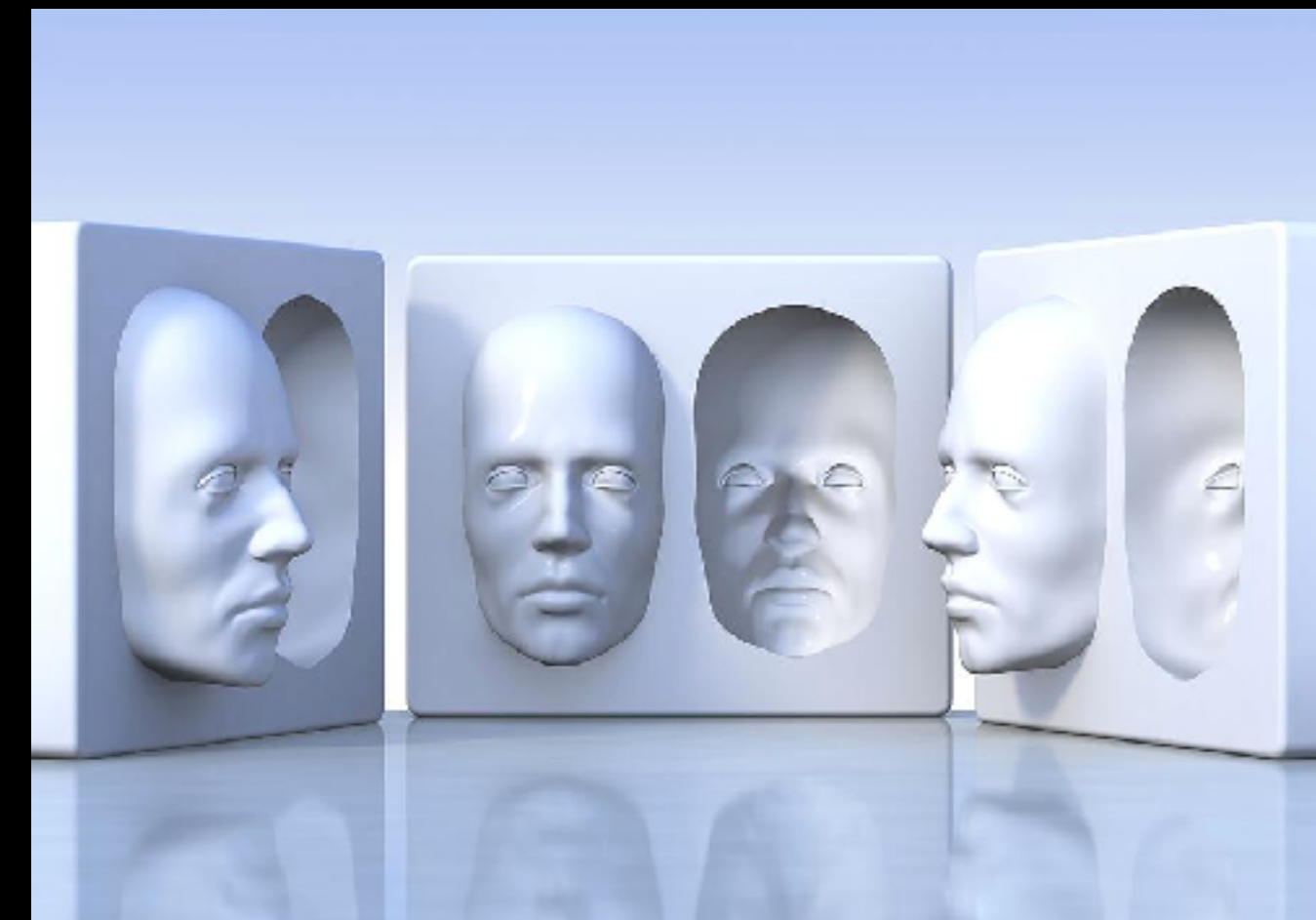
$$P_t^L = \frac{1}{\Omega} \mathcal{N}(s_t; \theta_t = \theta_L, \sigma^2) \left[(1-p_s) P_{t-1}^L + p_s (1 - P_{t-1}^L) \right]$$



*see Wei Ji's tutorial
from Cosyne 2019



Kersten & Yuille 2003

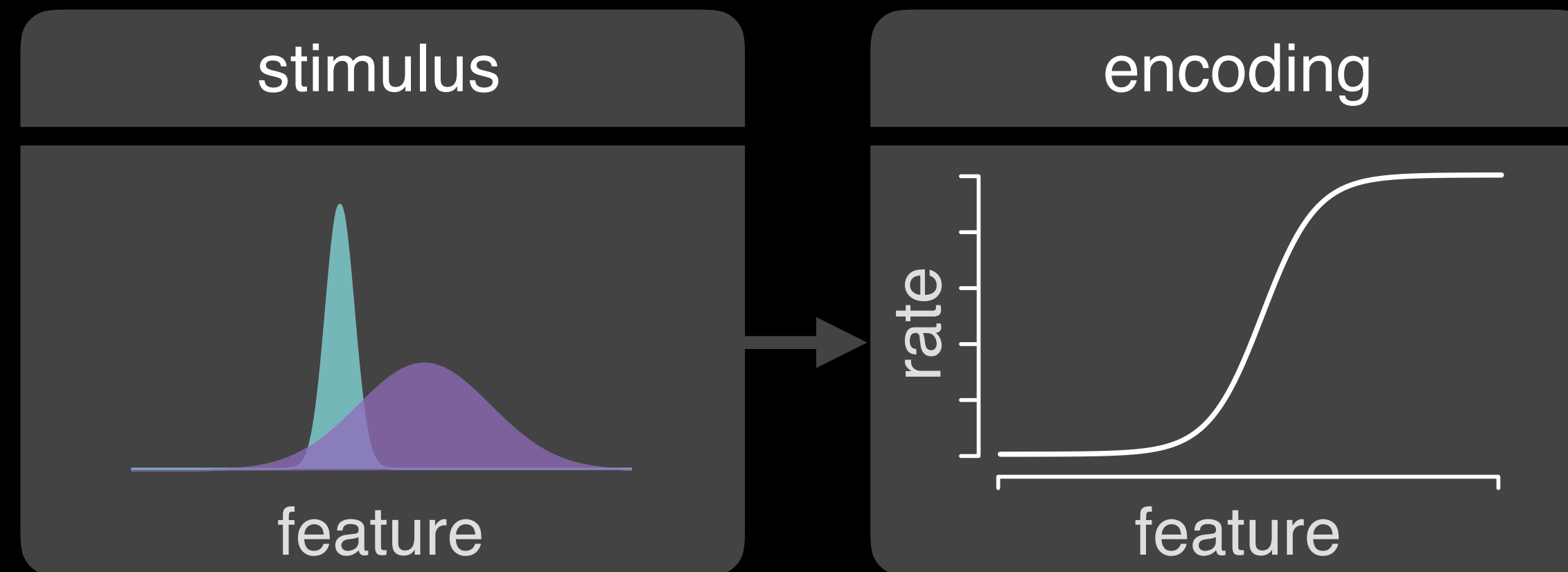


David Mack

$$P_t^L = \frac{1}{\Omega} \mathcal{N}(s_t; \theta_t = \theta_L, \sigma^2) \left[(1-p_s) P_{t-1}^L + p_s (1 - P_{t-1}^L) \right]$$

encoding

what features
should be prioritized
to maximize information?



$$P_t^L = \frac{1}{\Omega} \mathcal{N}(s_t; \theta_t = \theta_L, \sigma^2) \left[(1-p_s) P_{t-1}^L + p_s (1 - P_{t-1}^L) \right]$$

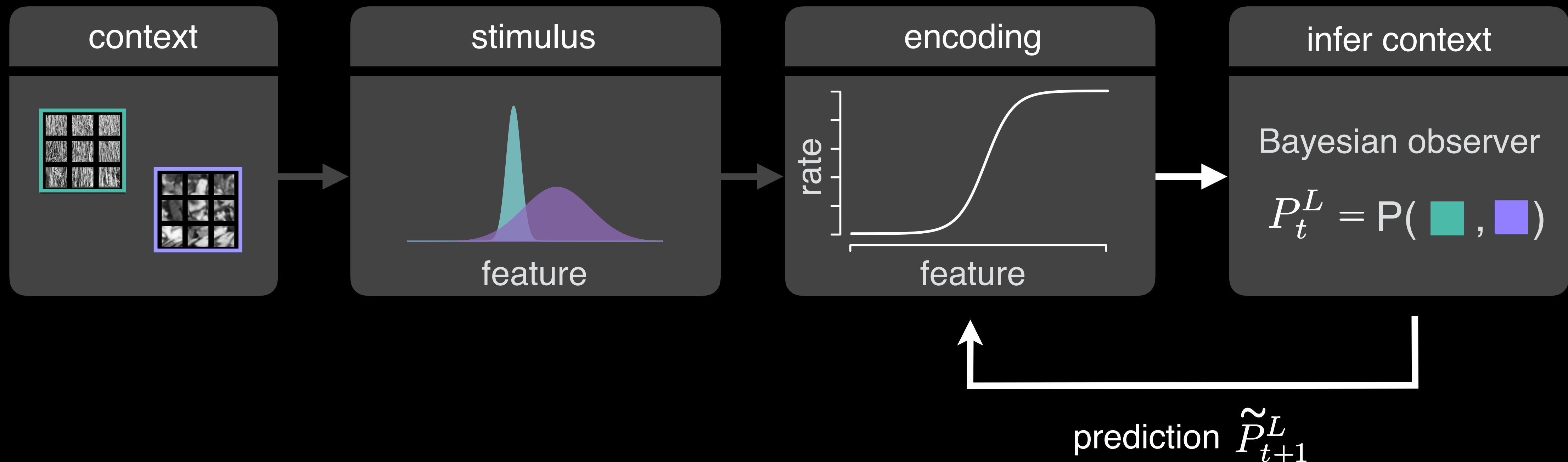
encoding



posterior

what features
should be prioritized
to maximize information?

how probable
are your hypotheses
about state of the world?



PART 1

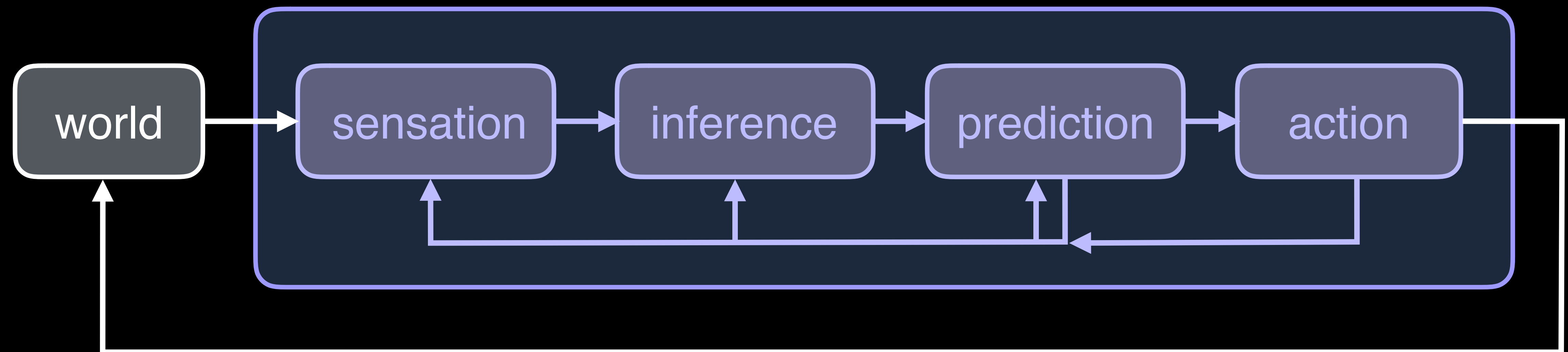
remove redundancy
combat noise

PART 2

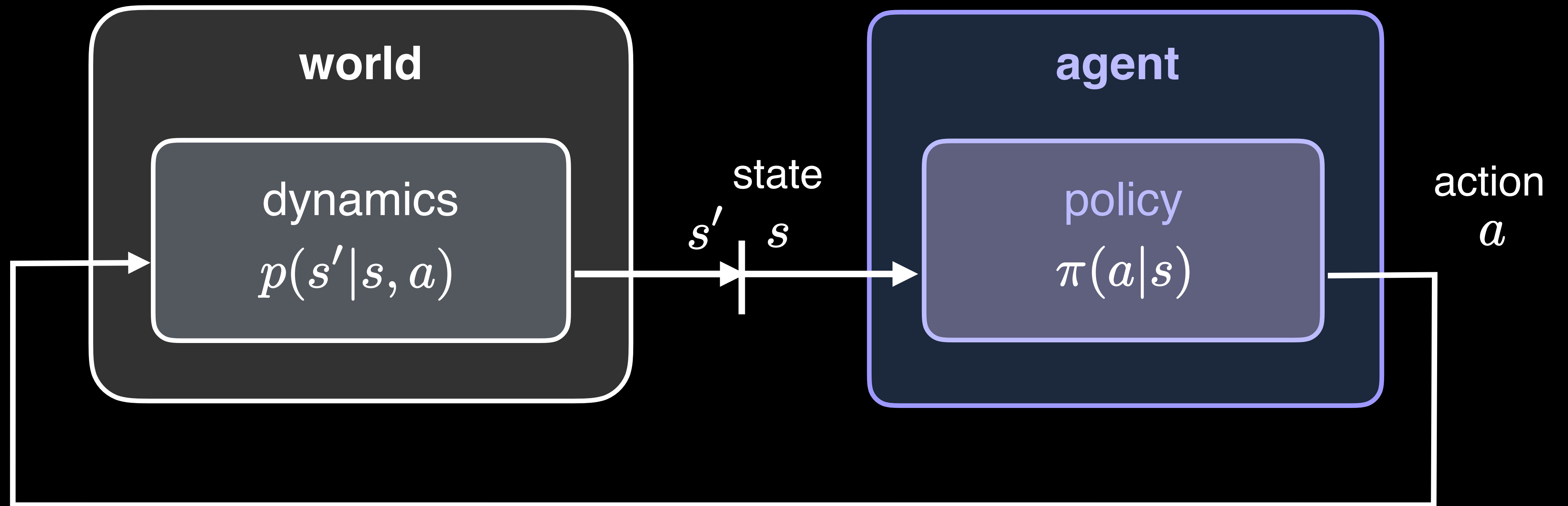
resolve ambiguity

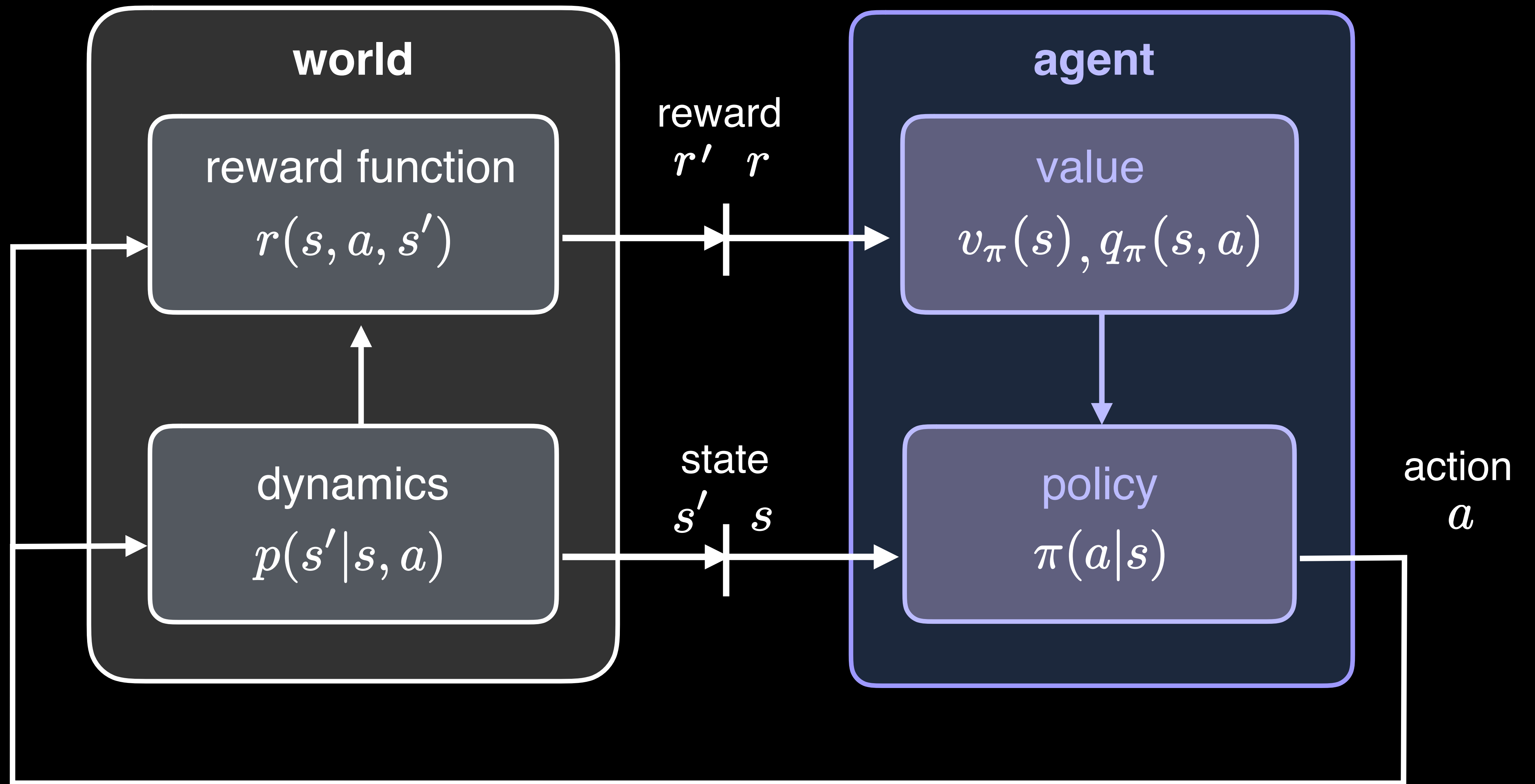
PART 3

make & use
predictions

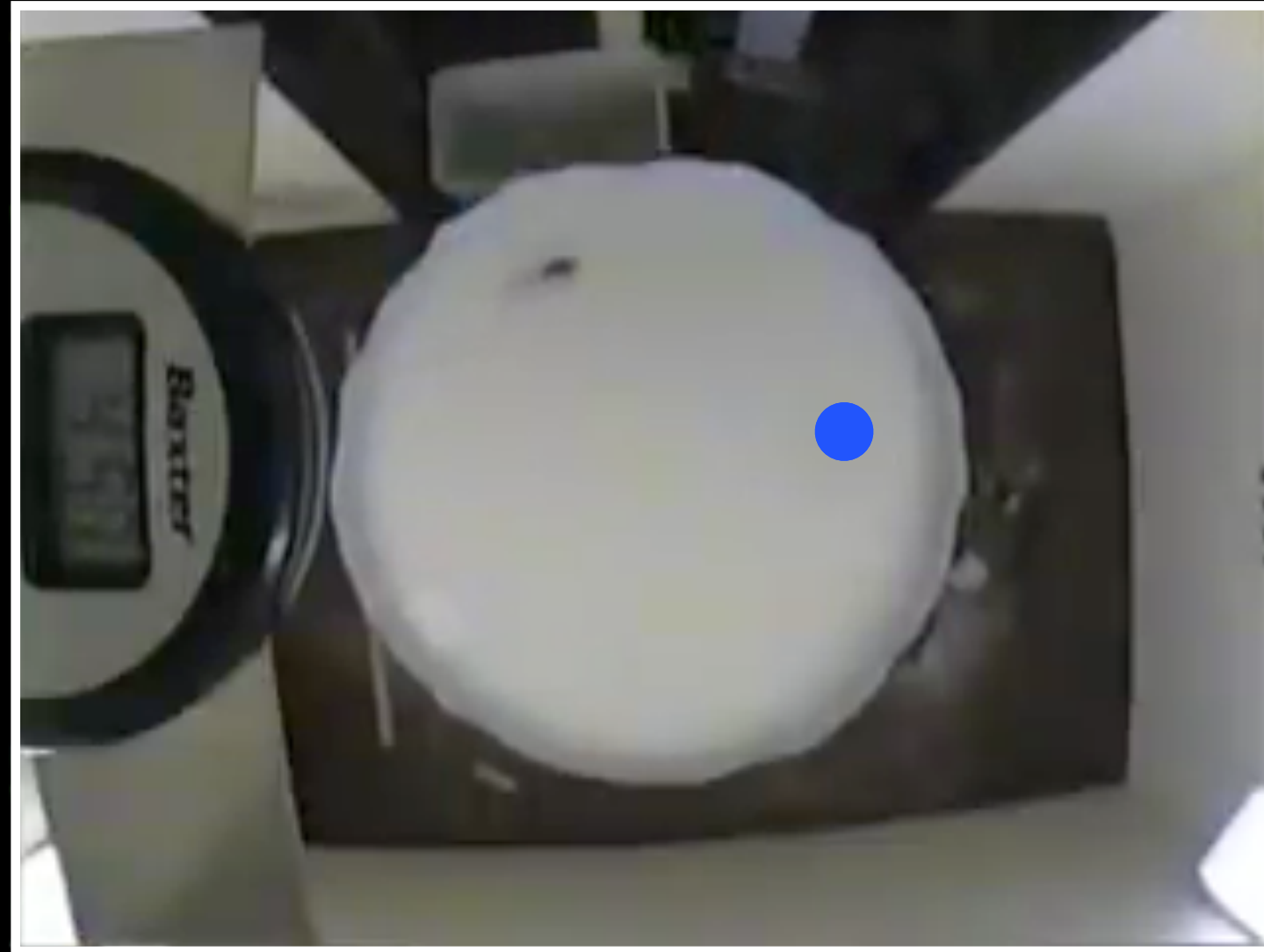


gather information (e.g. infotaxis)
maximize reward (e.g. reinforcement learning)





day 1, trial 1



day 5, trial 10

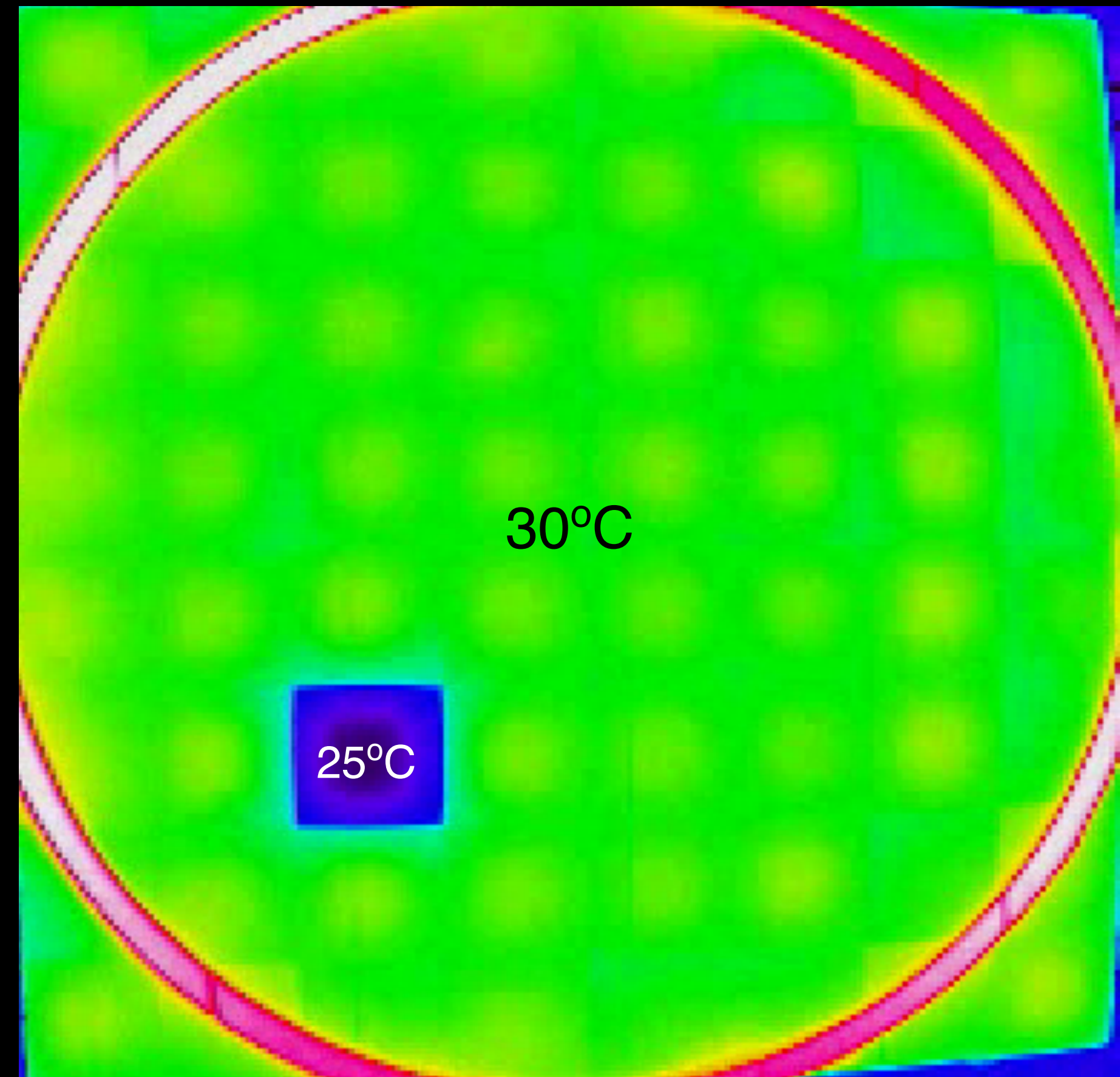
<https://tinyurl.com/vfwo8ze>

day 1, trial 1



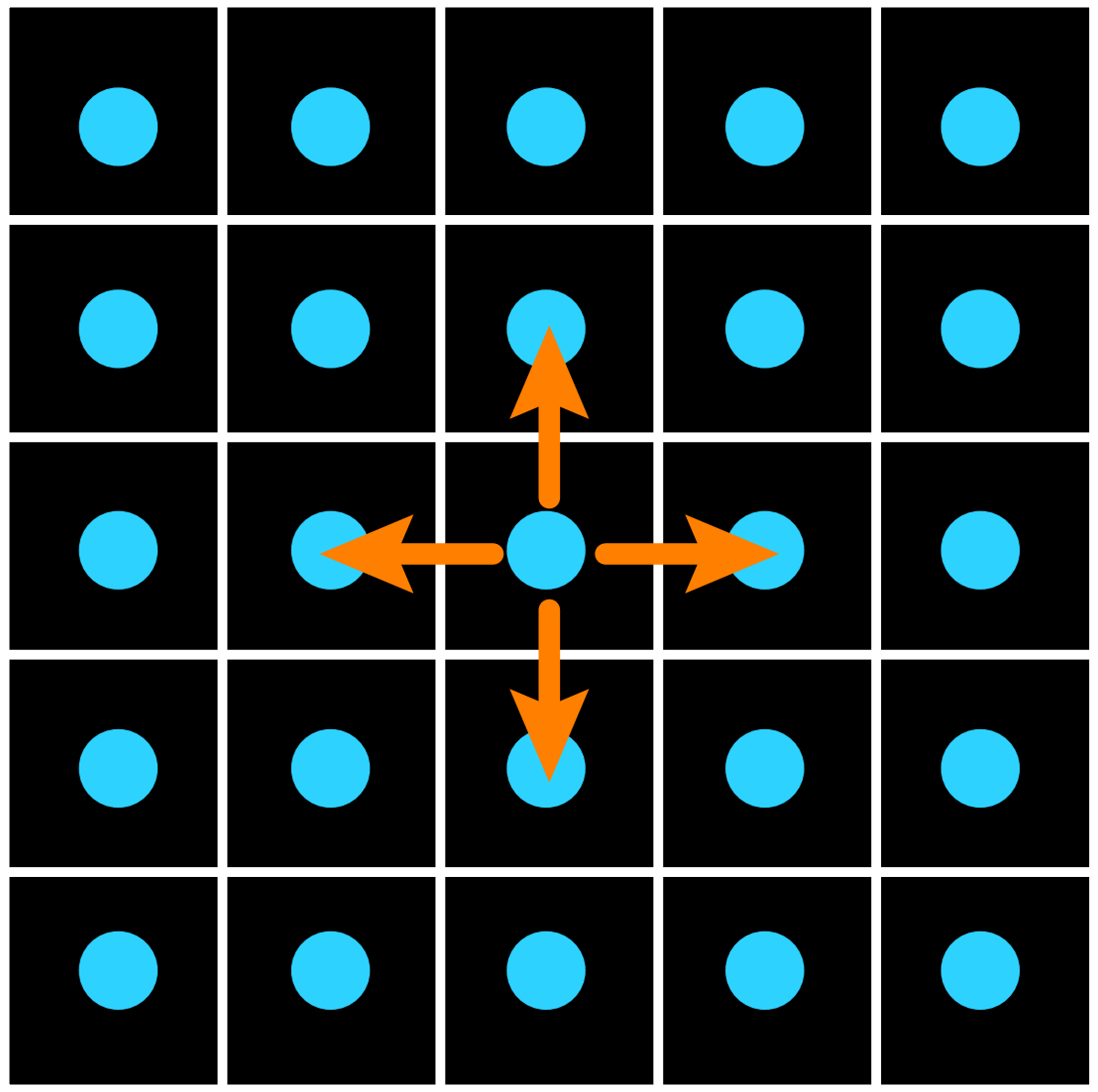
day 5, trial 10

<https://tinyurl.com/vfwo8ze>



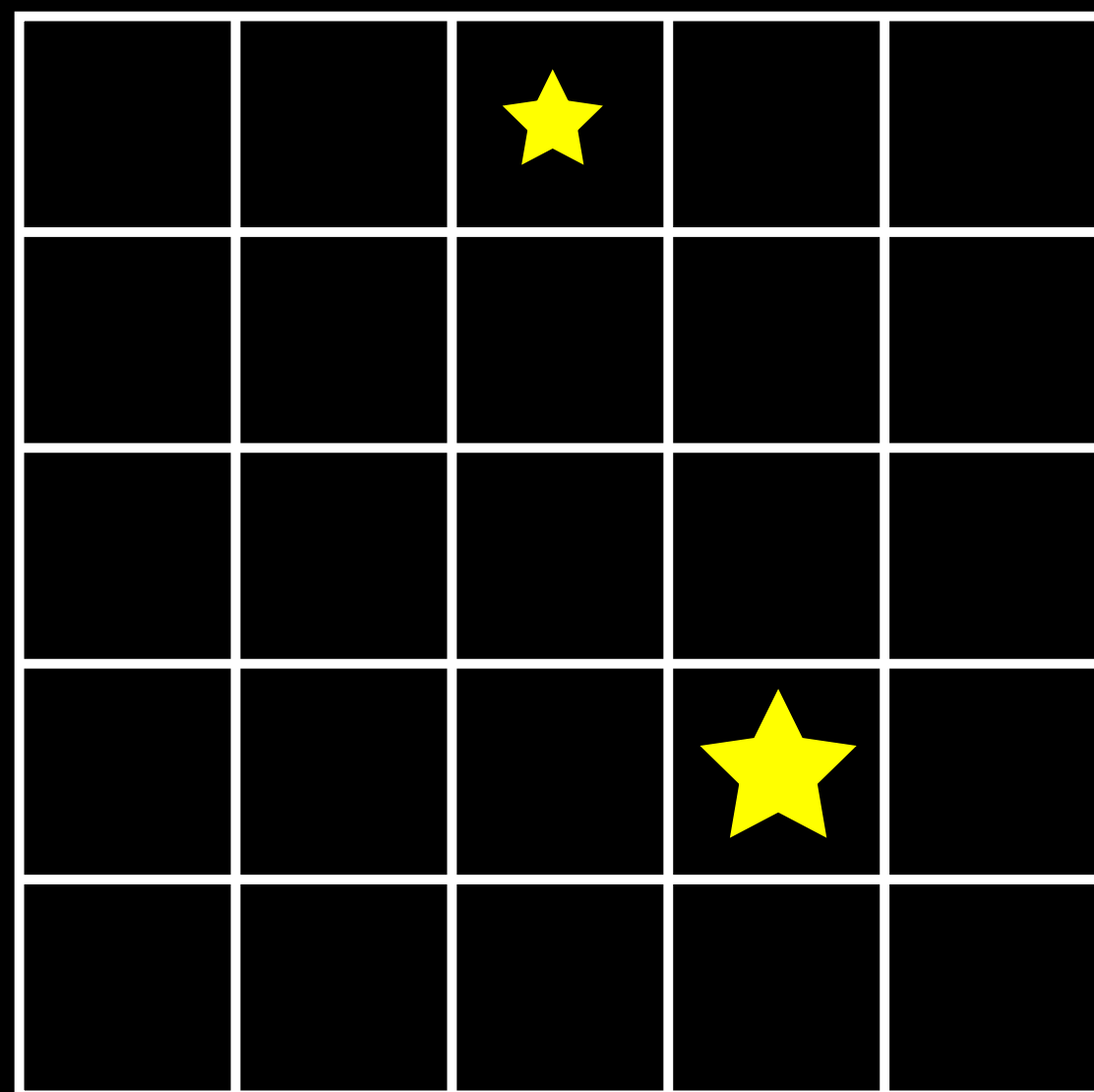
Ofstad, Zuker & Reiser, Nature (2011)

* fly tracking by Ctrax (Branson et al. 2009)

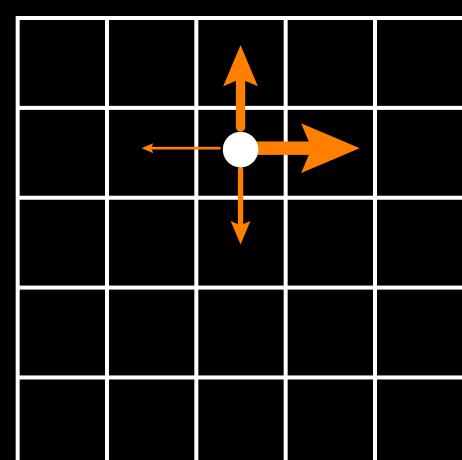


states s

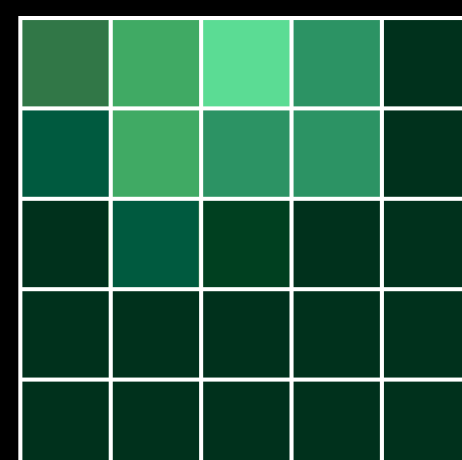
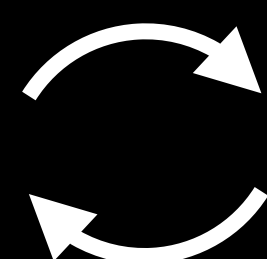
actions a



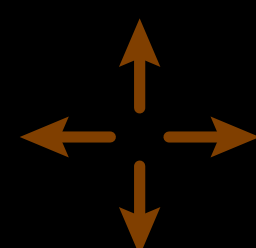
policy
 $\pi(a|s)$



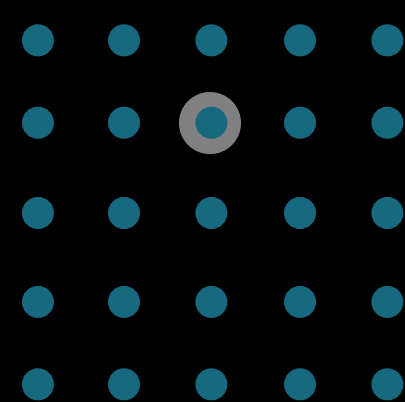
value $v_{\pi}(s)$
 $q_{\pi}(s, a)$

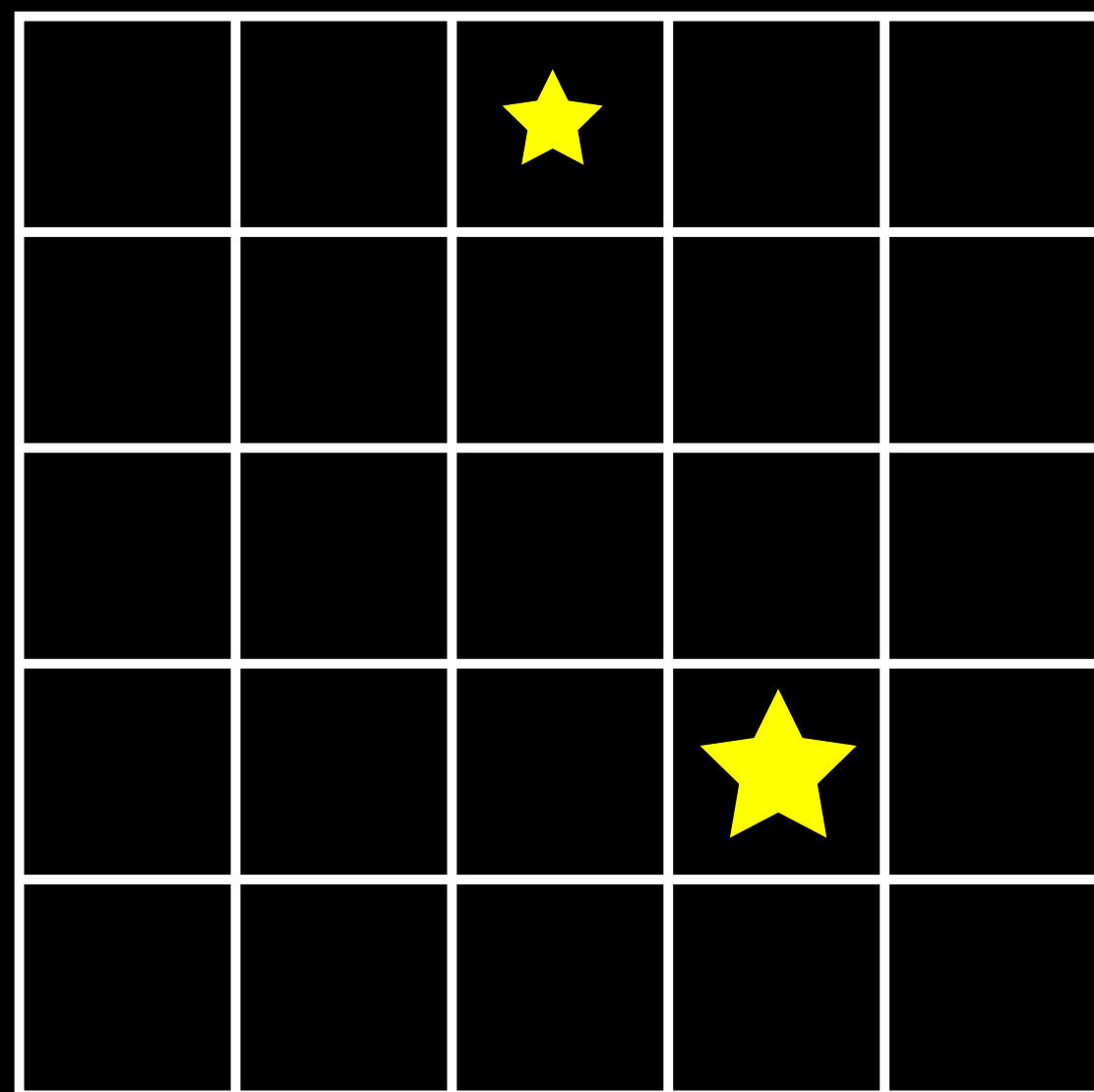


actions a



states s





explore / exploit tradeoff

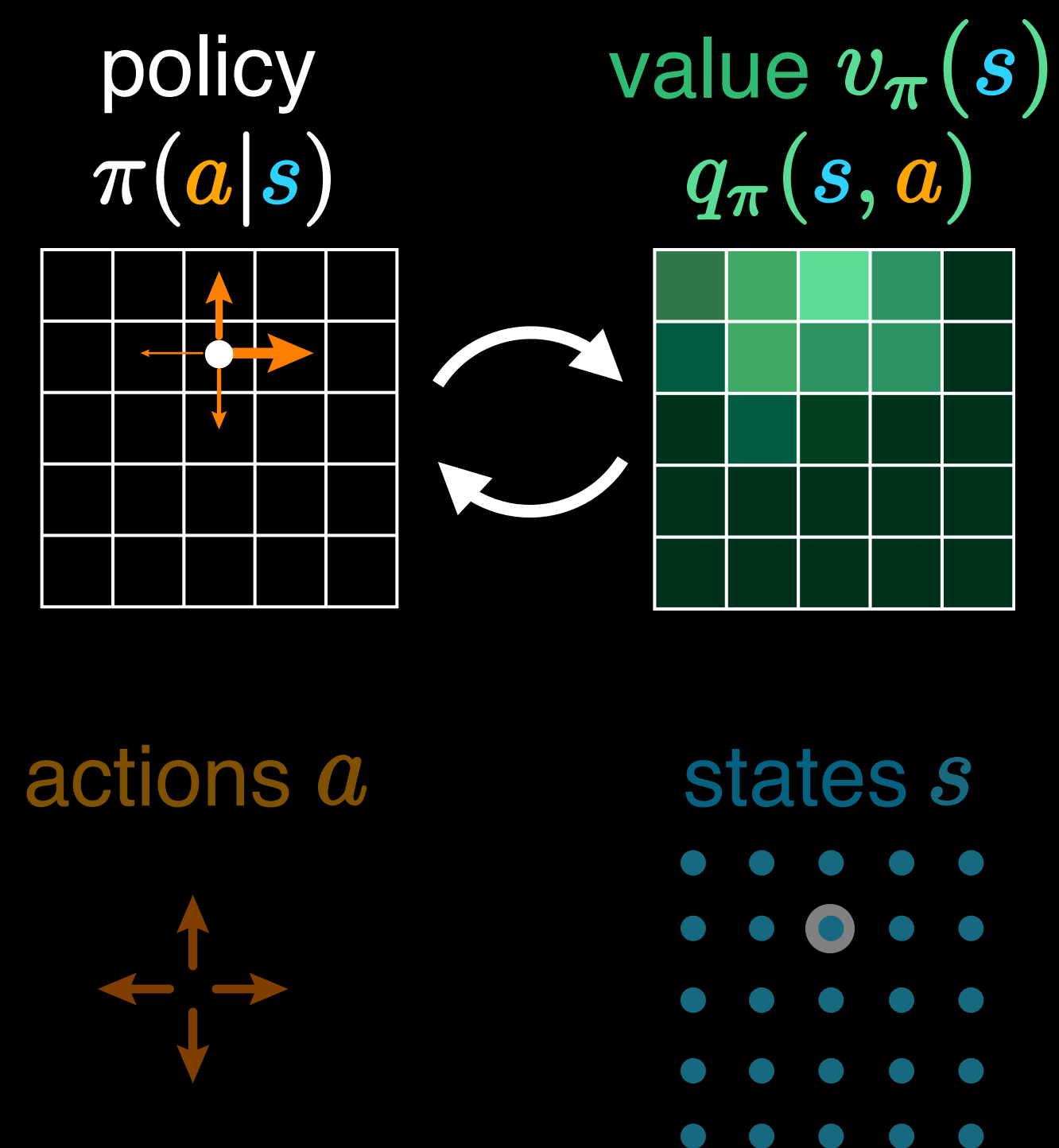
exploit: take action that gives highest expected value

explore: take action that has lower expected value but could result in higher long-term payoff

greedy: $A = \underset{a}{\operatorname{argmax}} q_{\pi}(s, a)$ exploit

ϵ -greedy: $(1 - \epsilon) \quad A = \text{greedy}$ exploit
 $\epsilon \quad A = \text{random}$ explore

softmax: $\pi(a|s) \propto \exp(\beta q_{\pi}(s, a))$
 $\beta \rightarrow \infty$ exploit
 $\beta \rightarrow 0$ explore



goal: maximize expected long-term reward
return G

$$G_t = \text{return, starting at time } t \\ = R_{t+1} + R_{t+2} + R_{t+3} + \dots$$

all rewards equally important

goal: maximize expected long-term reward
return G

G_t = return, starting at time t

$$= R_{t+1} + R_{t+2} + R_{t+3} + \dots$$

all rewards equally important

$$= R_{t+1} + R_{t+2} + R_{t+3} + \dots$$

goal: maximize expected long-term reward
return G

G_t = return, starting at time t

$$= R_{t+1} + R_{t+2} + R_{t+3} + \dots$$

all rewards equally important

$$= R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots$$

current rewards more important than distant ones

discount factor $\gamma \in [0, 1]$

$\gamma = 1$ don't discount (far sighted)

$\gamma = 0$ fully discount (myopic)

goal: maximize expected long-term reward
return G

G_t = return, starting at time t

$$= R_{t+1} + R_{t+2} + R_{t+3} + \dots$$

all rewards equally important

$$= R_{t+1} + \gamma \underbrace{\left[R_{t+2} + \gamma R_{t+3} + \dots \right]}_{G_{t+1}}$$

current rewards more important than distant ones

discount factor $\gamma \in [0, 1]$

$$G_t = R_{t+1} + \gamma G_{t+1}$$

$\gamma = 1$ don't discount (far sighted)

$\gamma = 0$ fully discount (myopic)

G_t represents actual future rewards (unknown to agent)

can instead compute expected future rewards, starting in state s , following policy π

consider one timestep in the future:

$$\mathbb{E}_{\pi} [R_{t+1} | S_t = s] = \sum_a \underbrace{\pi(a|s)}_{\text{policy}} \sum_{s'} \underbrace{p(s'|s, a)}_{\text{dynamics of environment}} \underbrace{r(s, a, s')}_{\text{reward function}}$$

$$G_t = R_{t+1} + \gamma G_{t+1}$$

define state-value function, starting in state s , following policy π

$$v_{\pi}(s) \equiv \mathbb{E}_{\pi} \left[G_t \mid S_t = s \right] \quad \text{prediction of rewards to come}$$

$$G_t = R_{t+1} + \gamma G_{t+1} \quad \mathbb{E}_{\pi} [R_{t+1} | S_t = s] = \sum_a \pi(a|s) \sum_{s'} p(s'|s, a) r(s, a, s')$$

define state-value function, starting in state s , following policy π

$$v_{\pi}(s) \equiv \mathbb{E}_{\pi} \left[\boxed{R_{t+1}} + \gamma G_{t+1} \mid S_t = s \right]$$

$$\mathbb{E}_{\pi} [R_{t+1} \mid S_t = s] = \sum_a \pi(a \mid s) \sum_{s'} p(s' \mid s, a) r(s, a, s')$$

define state-value function, starting in state s , following policy π

$$\begin{aligned} v_{\pi}(s) &\equiv \mathbb{E}_{\pi} \left[R_{t+1} + \gamma G_{t+1} \mid S_t = s \right] \\ &= \sum_a \pi(a|s) \sum_{s'} p(s'|s, a) \left[r(s, a, s') \right. \\ &\quad \left. + \gamma \underbrace{\mathbb{E}_{\pi} \left[G_{t+1} \mid S_{t+1} = s' \right]}_{v_{\pi}(s')} \right] \end{aligned}$$

define state-value function, starting in state s , following policy π

$$v_{\pi}(s) \equiv \mathbb{E}_{\pi} \left[R_{t+1} + \gamma G_{t+1} \mid S_t = s \right]$$
$$= \sum_a \pi(a|s) \sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma v_{\pi}(s') \right]$$

Bellman equation
for state-value function

define state-value function, starting in state s , following policy π

$$v_{\pi}(s) = \sum_a \pi(a|s) \underbrace{\sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma v_{\pi}(s') \right]}_{q_{\pi}(s, a)} \quad \begin{array}{l} \text{state-value function} \\ \text{action-value function} \end{array}$$

Dynamic Programming:	given $p(s' s, a), r(s, a, s')$, learn optimal v_*, q_*, π_* via bootstrapping	} temporal difference (TD) learns from experience via bootstrapping
Monte Carlo:	estimate v_*, q_* via sampling, learn optimal π_* from simulated experiences	

define state-value function, starting in state s , following policy π

$$v_{\pi}(s) = \sum_a \pi(a|s) \sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma v_{\pi}(s') \right] \quad \text{state-value function}$$

temporal difference (TD) : improve estimate of value through experience

$$\begin{array}{ccccccc} \text{new} & & \text{old} & & \text{step} & & \\ \text{estimate} & = & \text{estimate} & + & \text{size} & \left[& \text{target} & - & \text{old} & \right] \\ & & & & & & & & \text{estimate} \end{array}$$

$$V_{t+1}(s) = V_t(s) + \alpha \left[\underline{G_t} - V_t(s) \right]$$
$$\simeq (R + \gamma V_t(s'))$$

define state-value function, starting in state s , following policy π

$$v_{\pi}(s) = \sum_a \pi(a|s) \sum_{s'} p(s'|s, a) \left[r(s, a, s') + \gamma v_{\pi}(s') \right] \quad \text{state-value function}$$

$$V_{t+1}(s) = V_t(s) + \alpha \left[\underbrace{(R + \gamma V_t(s'))}_{\text{prediction error } \delta_t} - V_t(s) \right]$$

$$Q_{t+1}(s, a) = Q_t(s, a) + \alpha \left[\underbrace{(R + \gamma Q_t(s', a'))}_{\text{prediction error } \delta_t} - Q_t(s, a) \right] \quad \text{Q-learning, SARSA}$$

here, states can only be updated as they are visited

to update states that were visited in the past, we can use eligibility traces

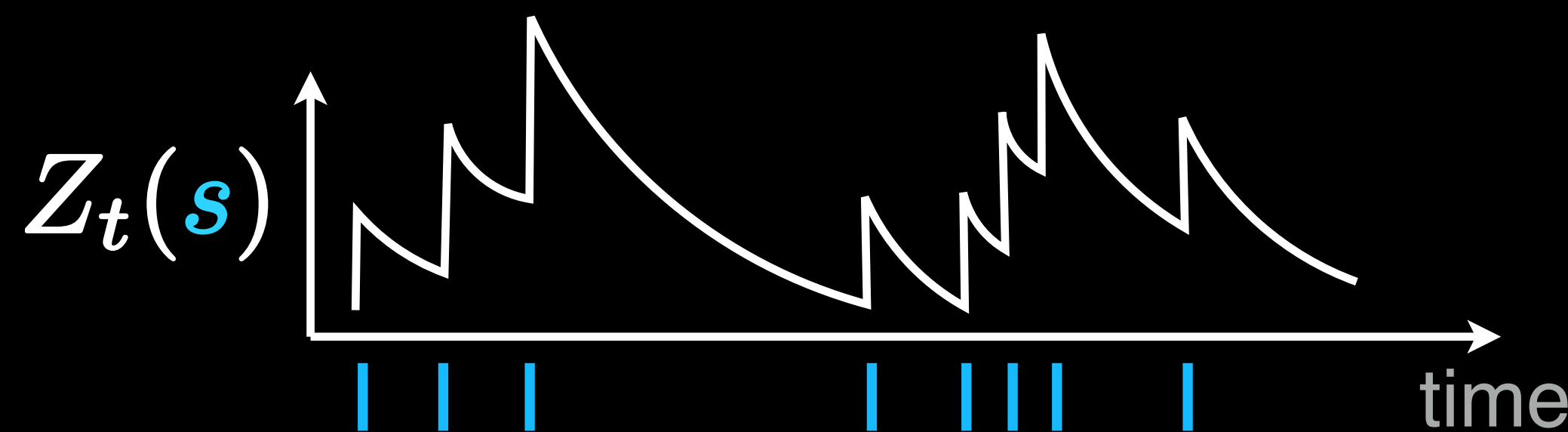
$$V_{t+1}(s) = V_t(s) + \alpha \left[\left(R + \gamma V_t(s') \right) - V_t(s) \right] \frac{Z_t(s)}{\text{“eligibility” of state } s}$$

$$Z_t(s) = \begin{cases} \lambda \gamma Z_{t-1}(s) & s \neq S_t \\ 1 + \lambda \gamma Z_{t-1}(s) & s = S_t \end{cases}$$

trace-decay parameter $\lambda \in [0, 1]$

$\lambda = 0$ only current state can be updated

$\lambda = 1$ eligibility falls by γ each timestep

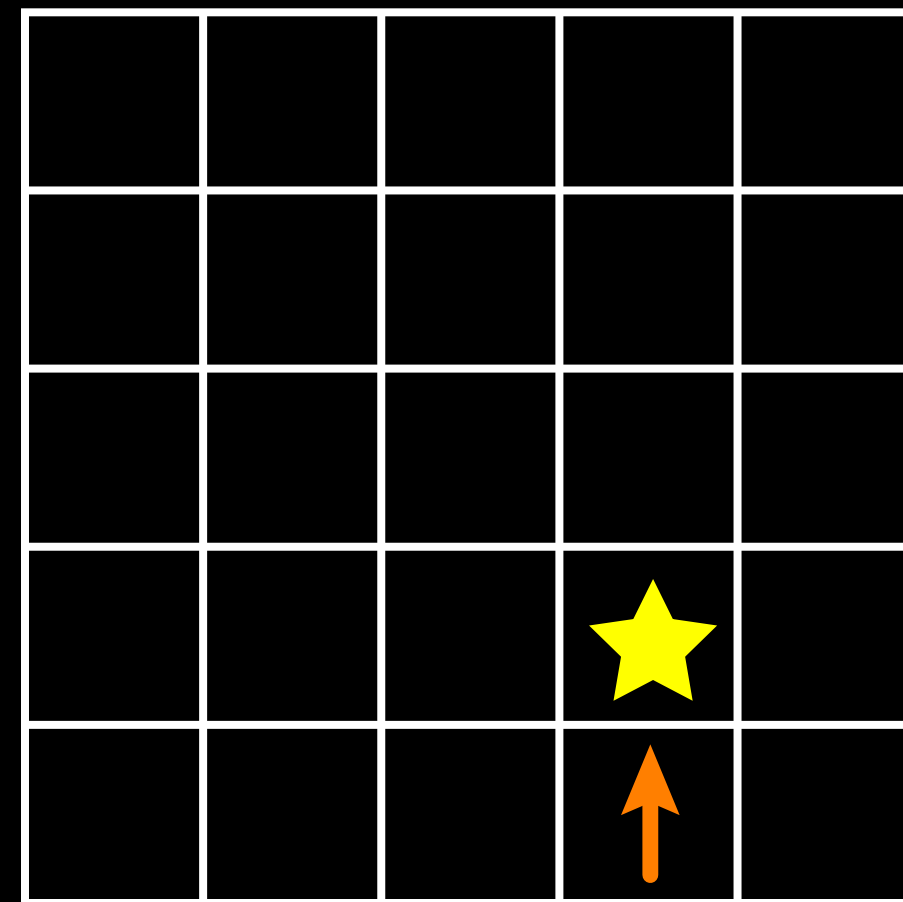
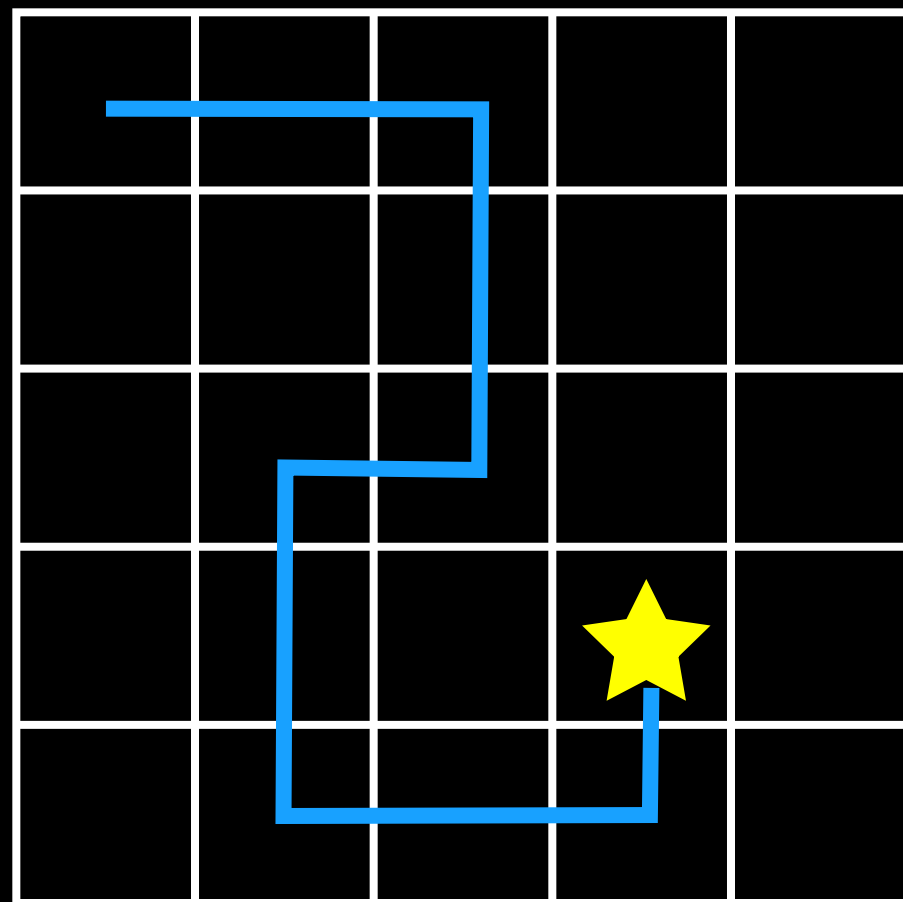


here, states can only be updated as they are visited

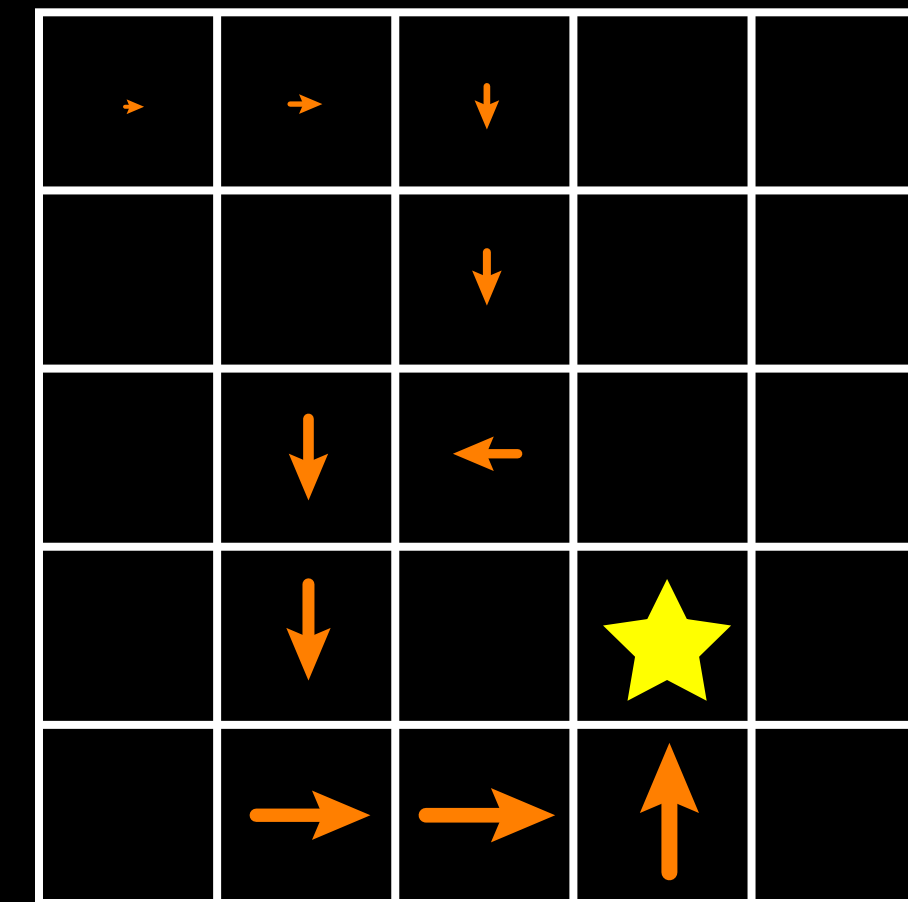
to update states that were visited in the past, we can use eligibility traces

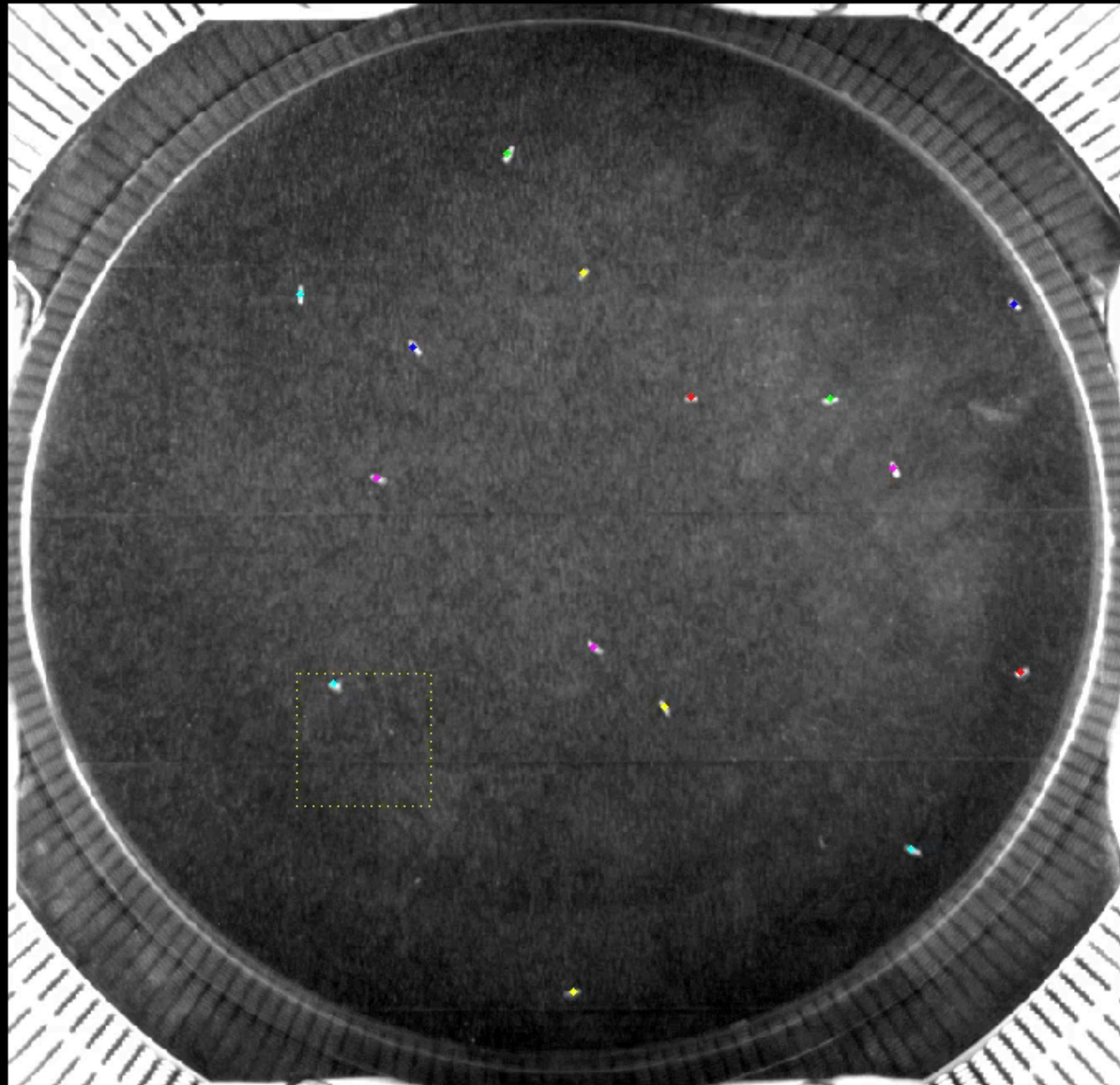
$$V_{t+1}(s) = V_t(s) + \alpha \left[\left(R + \gamma V_t(s') \right) - V_t(s) \right] \quad Z_t(s) = \begin{cases} \lambda \gamma Z_{t-1}(s) & s \neq S_t \\ 1 + \lambda \gamma Z_{t-1}(s) & s = S_t \end{cases}$$

$\lambda = 0$



$\lambda > 0$





Ofstad, Zuker & Reiser, Nature (2011)
 * fly tracking by Ctrax (Branson et al. 2009)

day 1, trial 1



day 5, trial 10

<https://tinyurl.com/vfwo8ze>

back at 2:40

wifi

Hilton Honors Meeting
csnventi20

problem set & code

<http://bit.ly/cosyne2020-tutorial>

worksheet

(handout)

outline

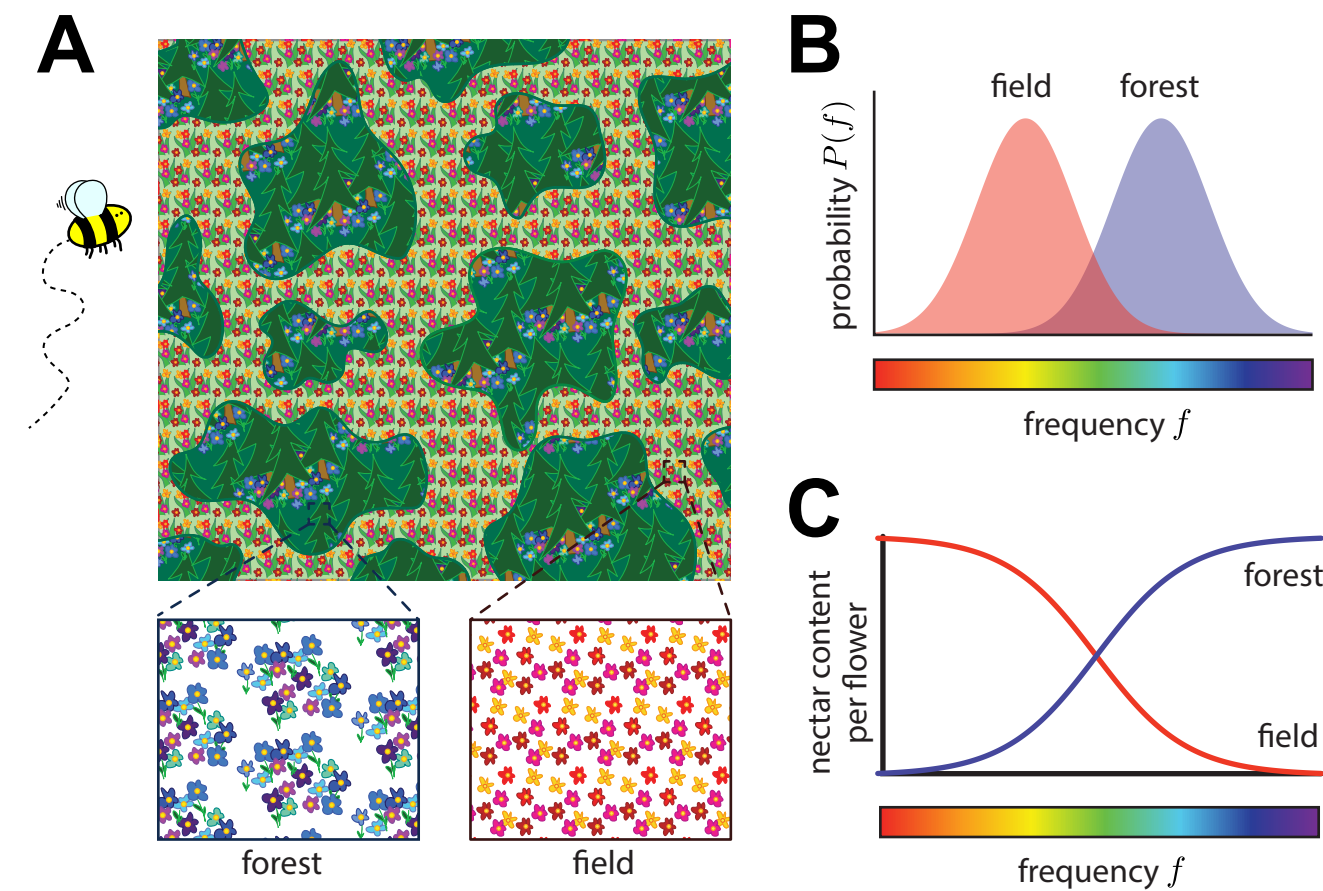
- 1 | problem setup
- 2 | sensory coding
- 3 | inference
- 4 | action selection

do these first!

* come back to these
after you've finished
all 3 sections

problem set

Problem Set: Cosyne 2020



2 Sensory Coding

⋮

2.3 What would you expect ...

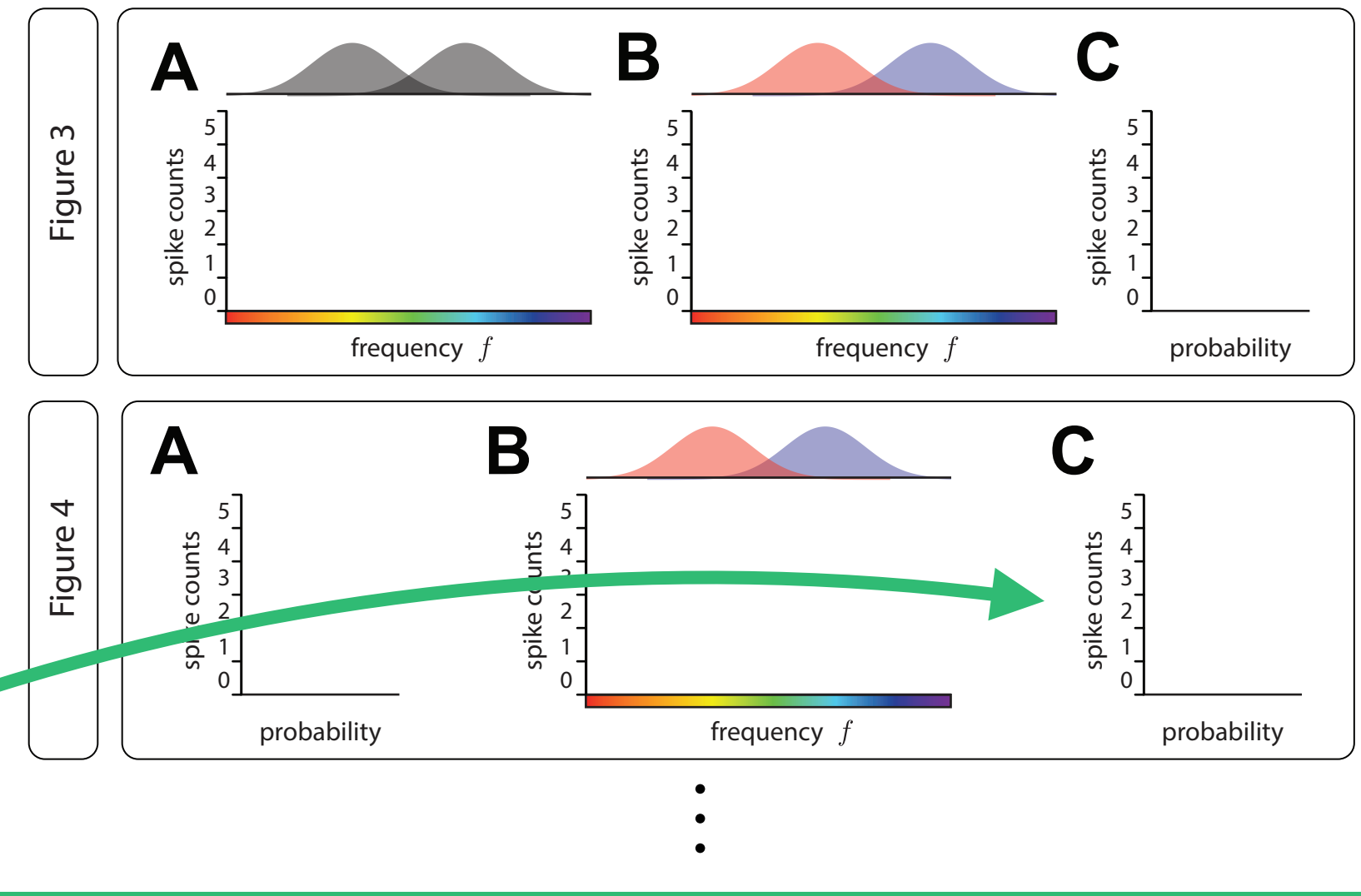
2.4 Use Fig 4C to sketch ...

*2.5 Parameterize the nonlinearity ...
Numerically, find the optimal ...

worksheet

Worksheet: Cosyne 2020

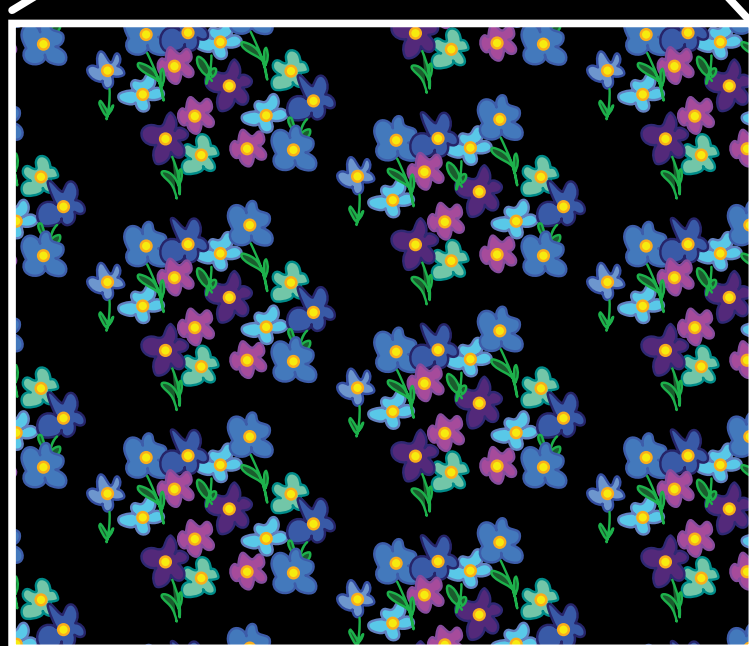
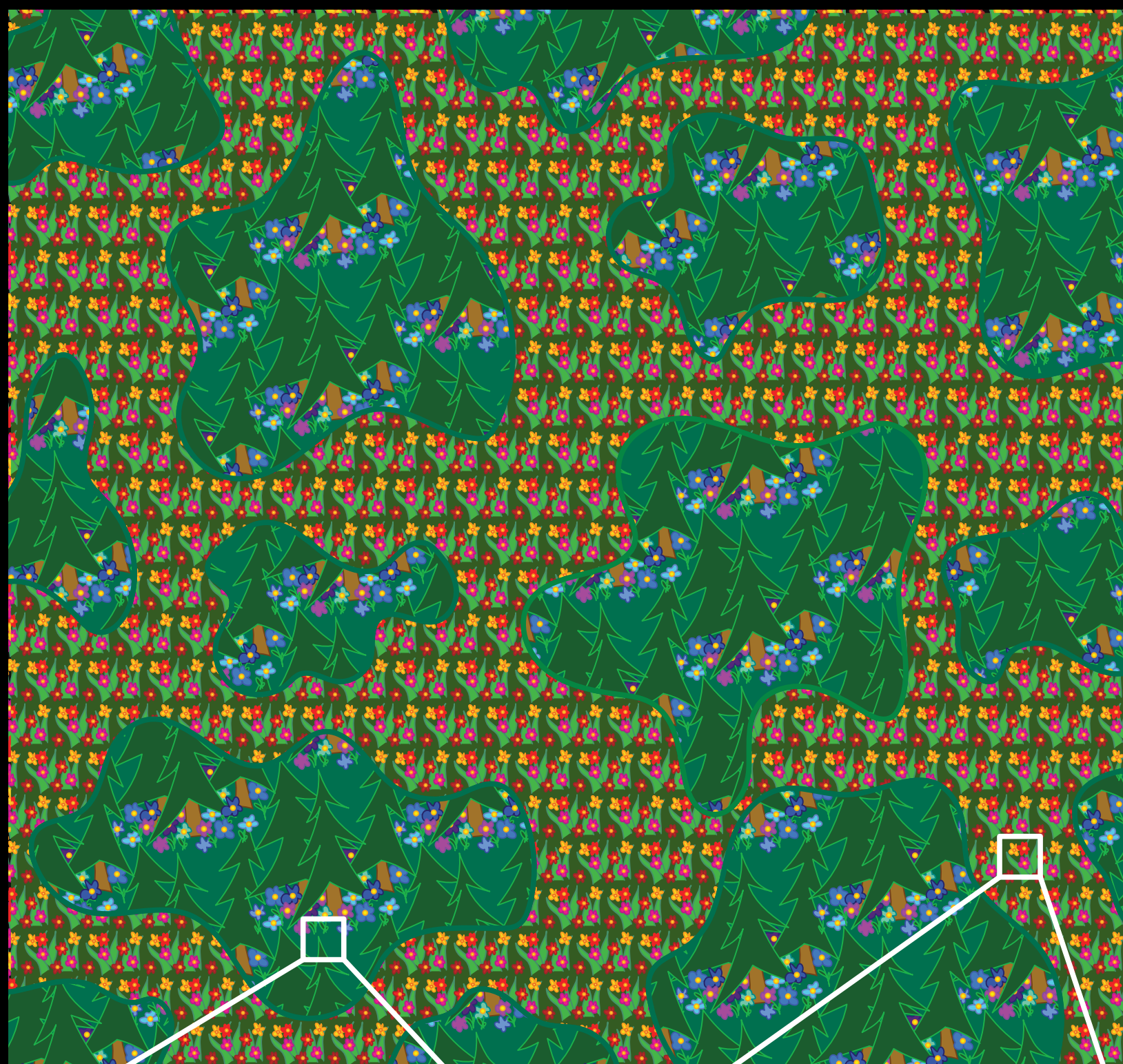
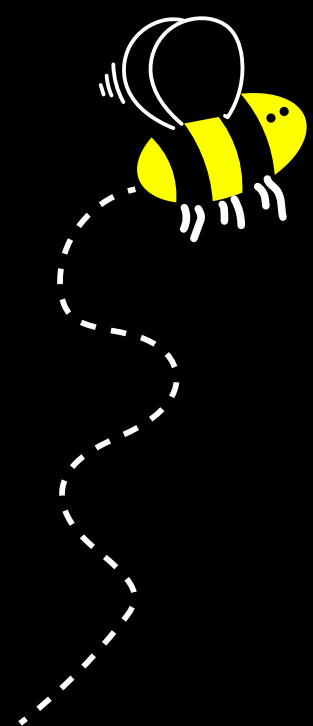
Ann Hermundstad (hermundstada@janelia.hhmi.org)



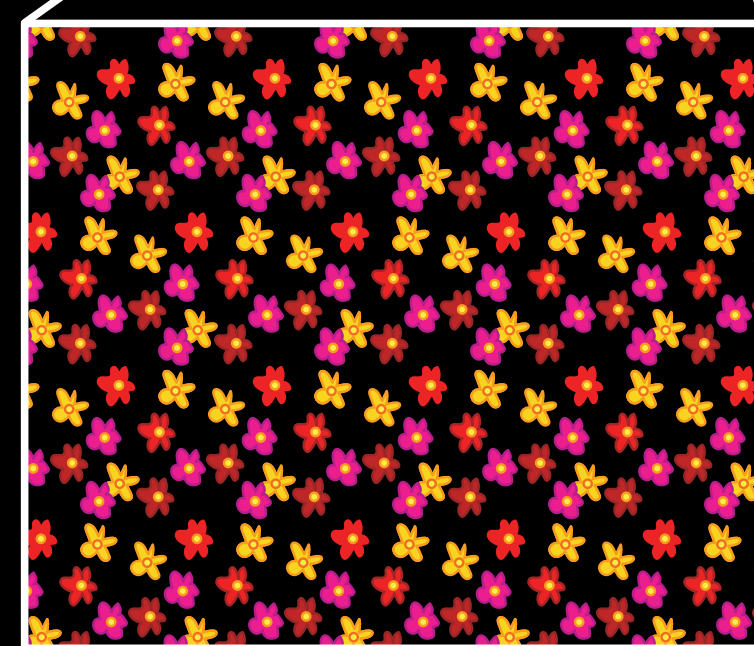
code available
(thanks to Sashank Pisupati!)

[http://bit.ly/
cosyne2020-tutorial](http://bit.ly/cosyne2020-tutorial)

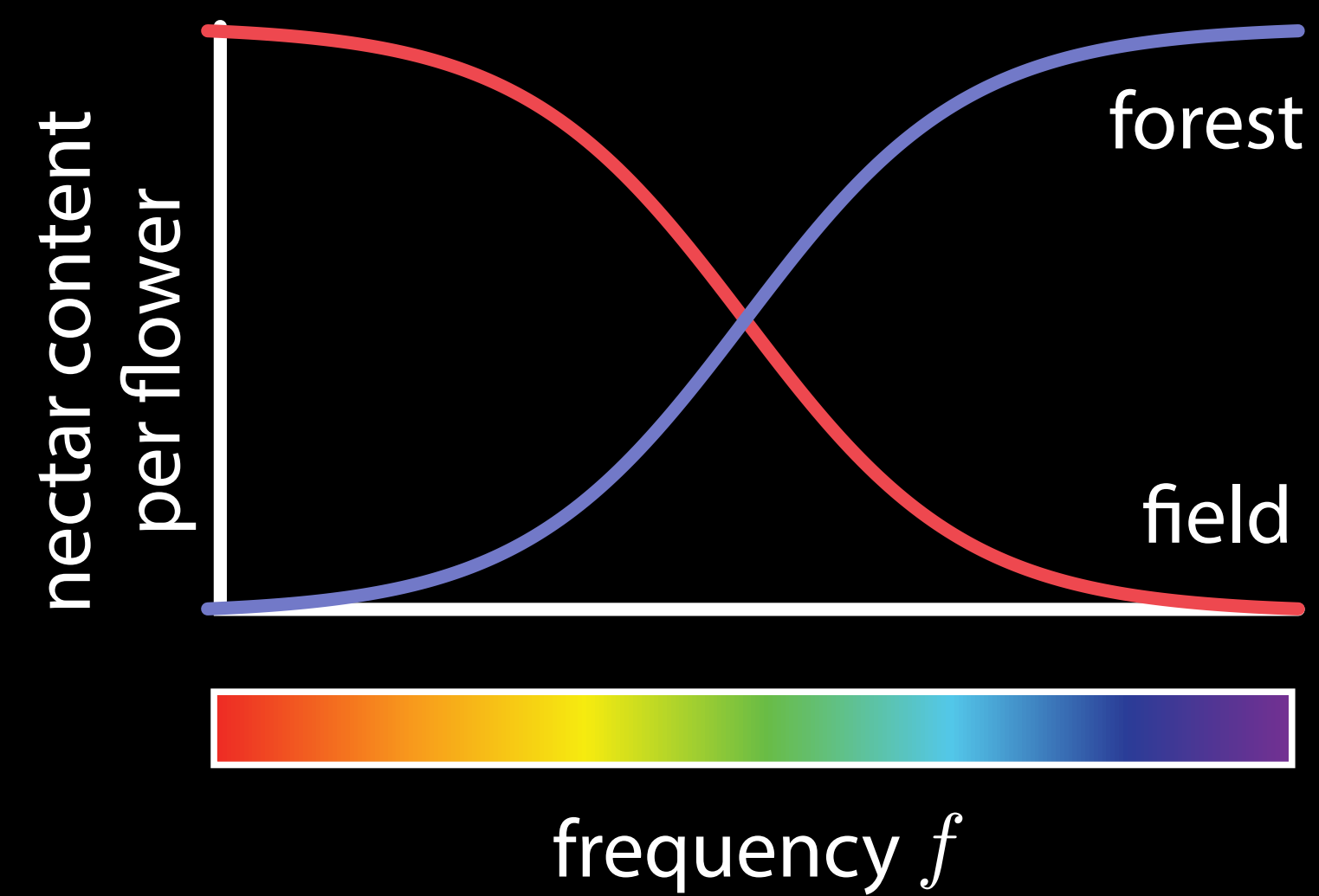
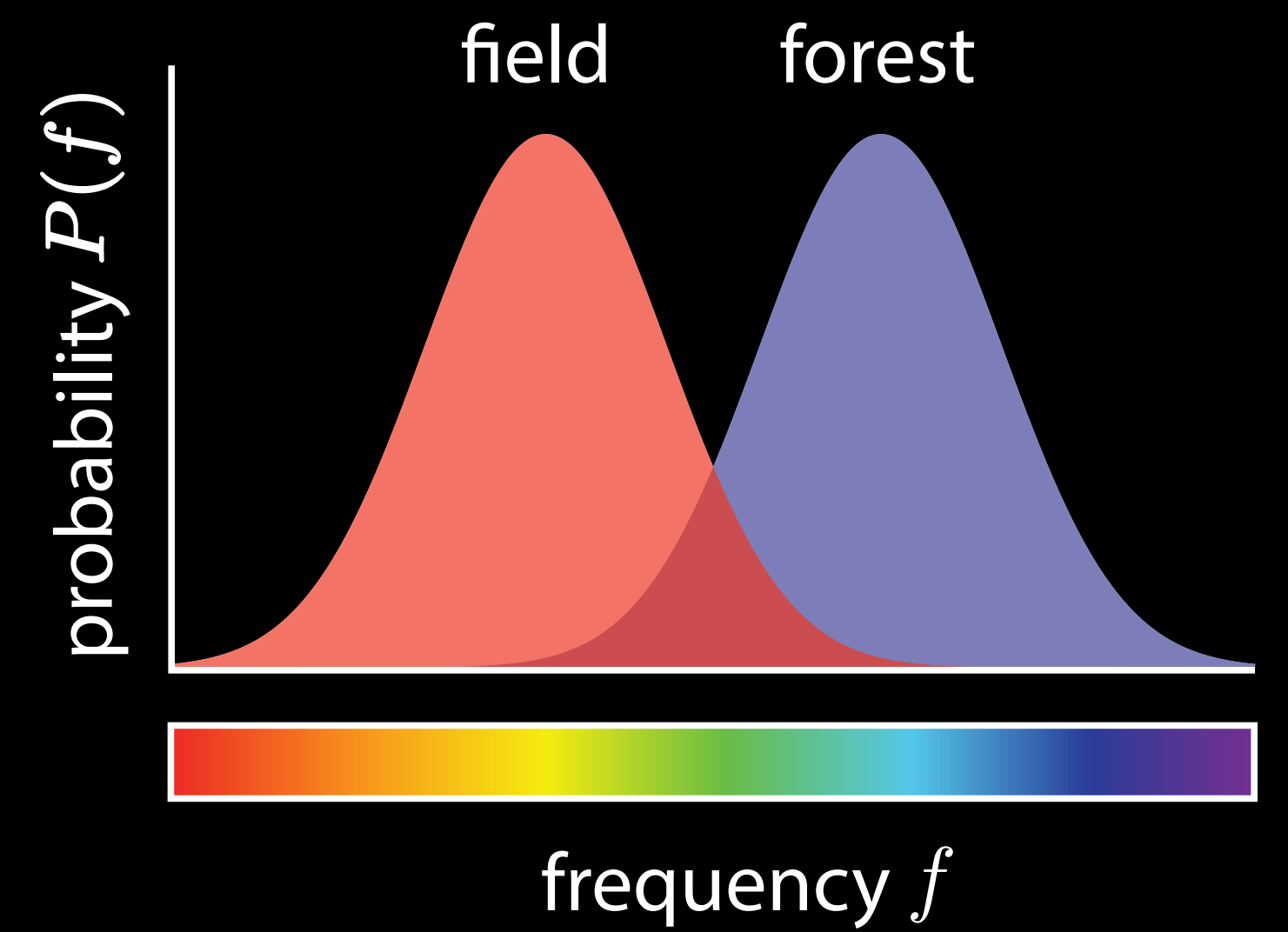
problem recap

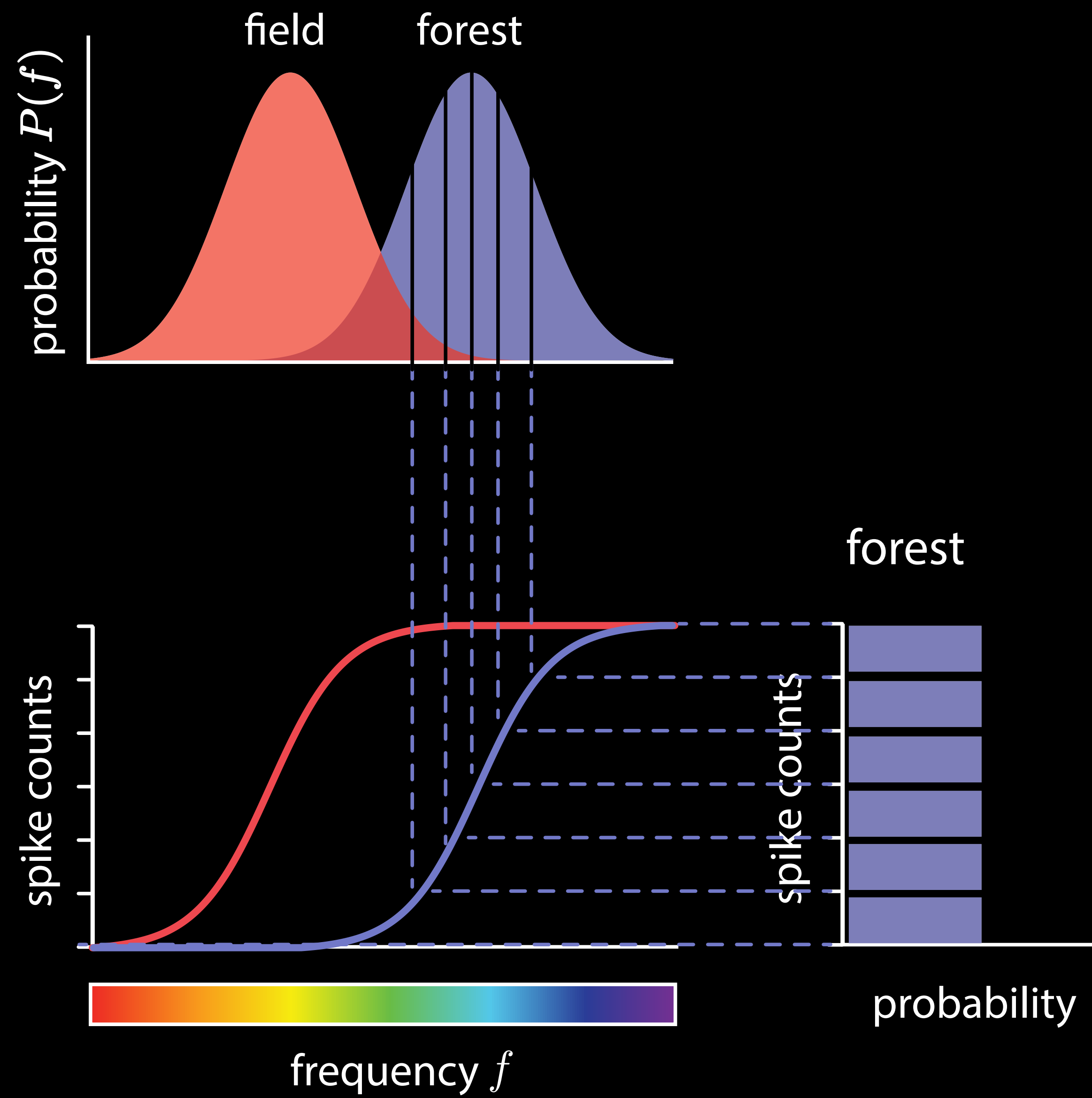
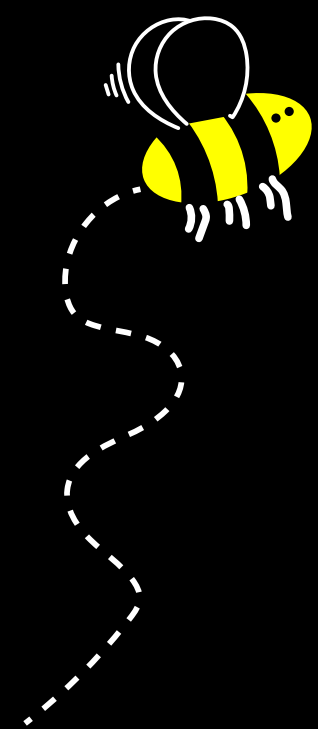


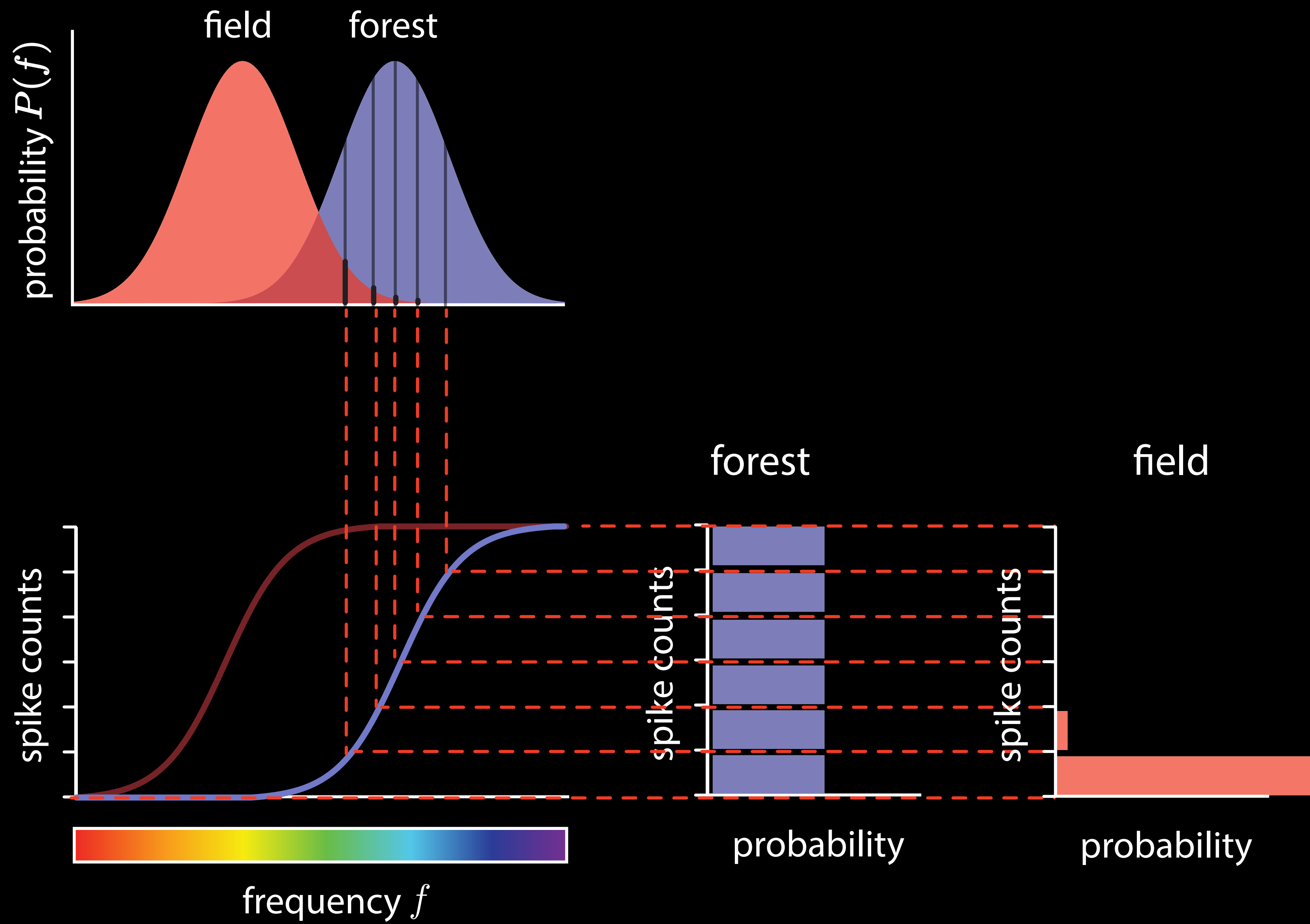
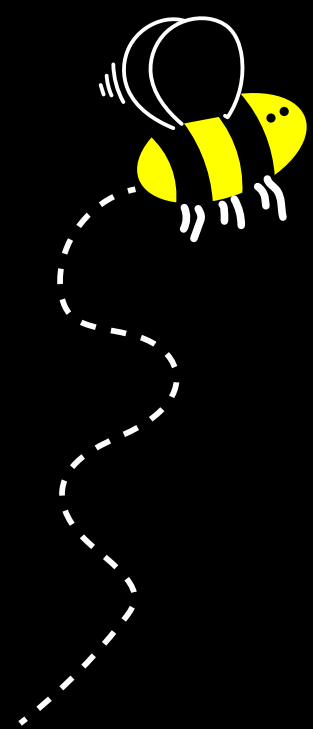
forest

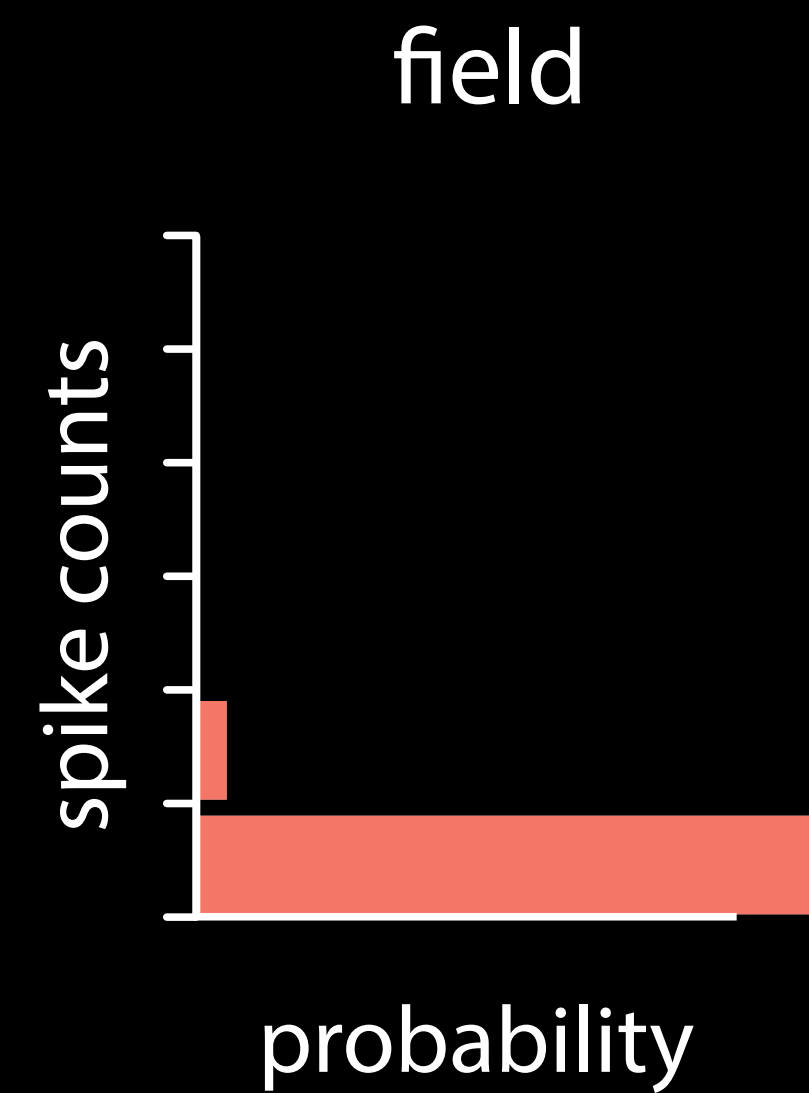
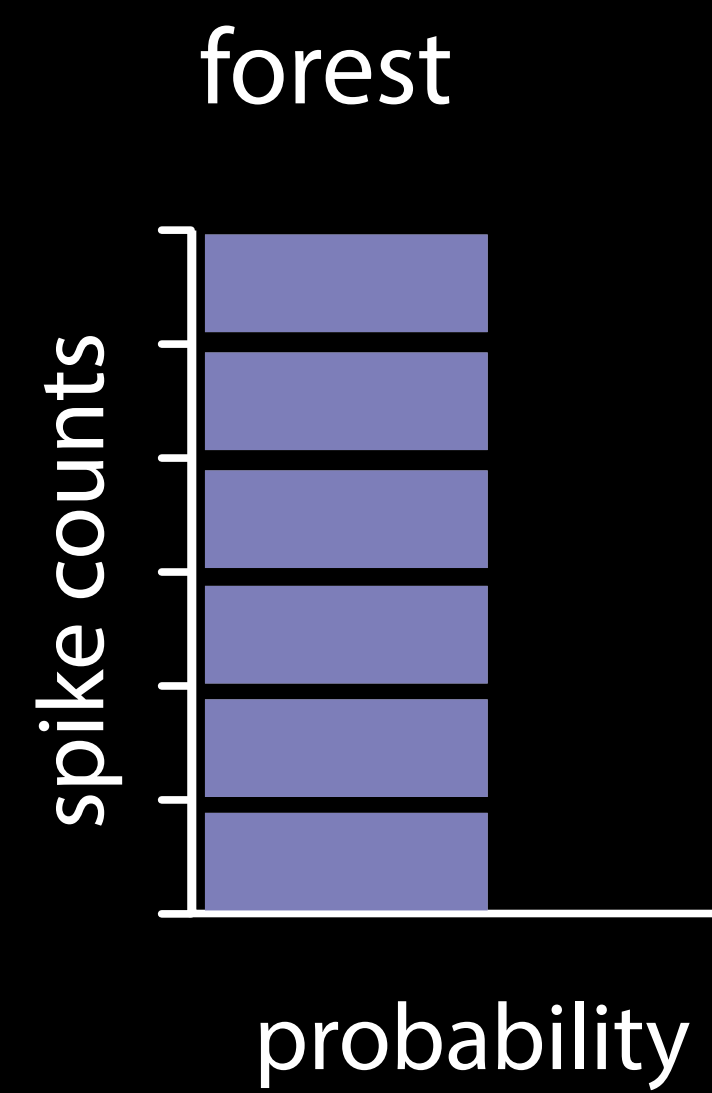
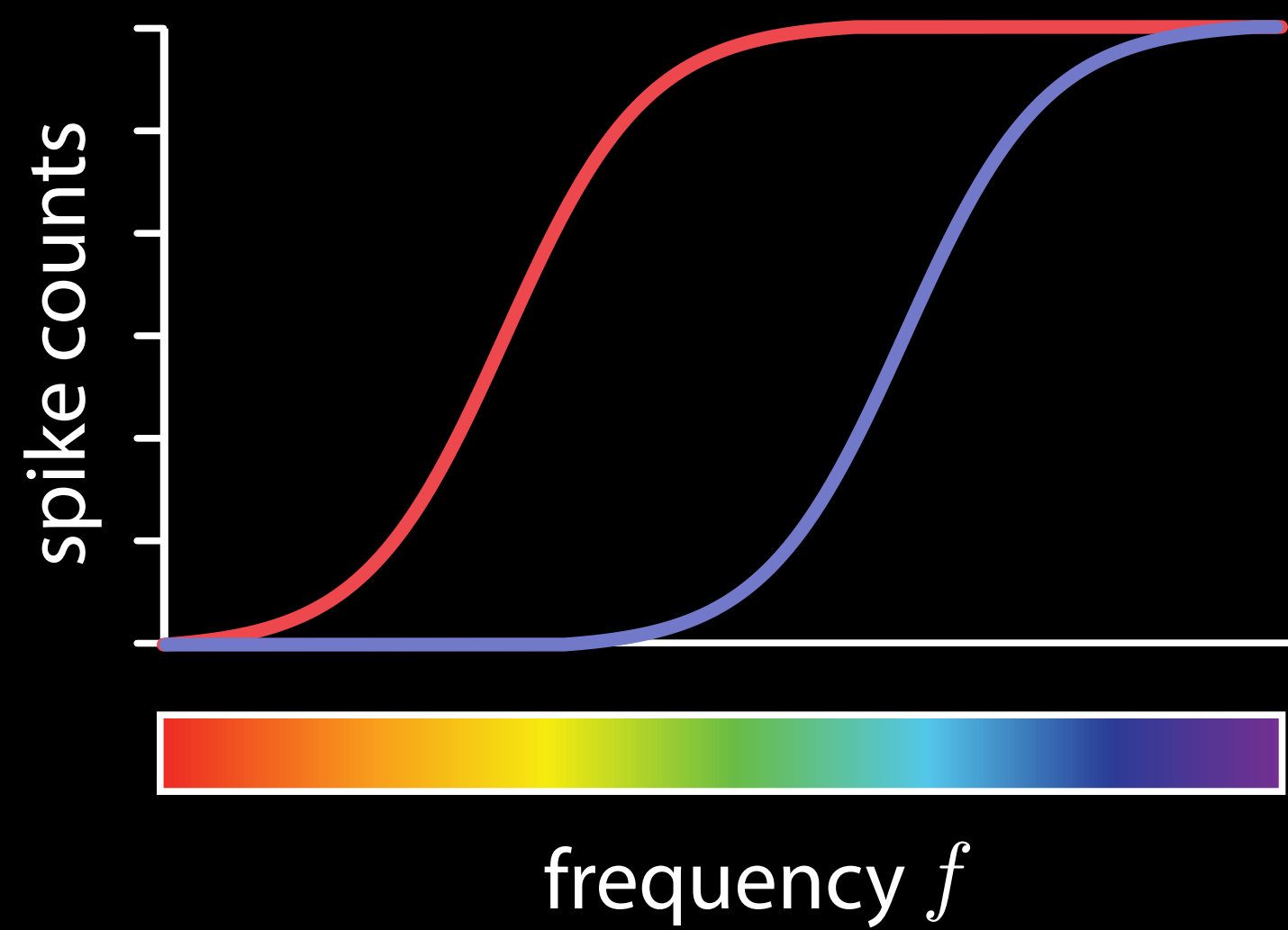
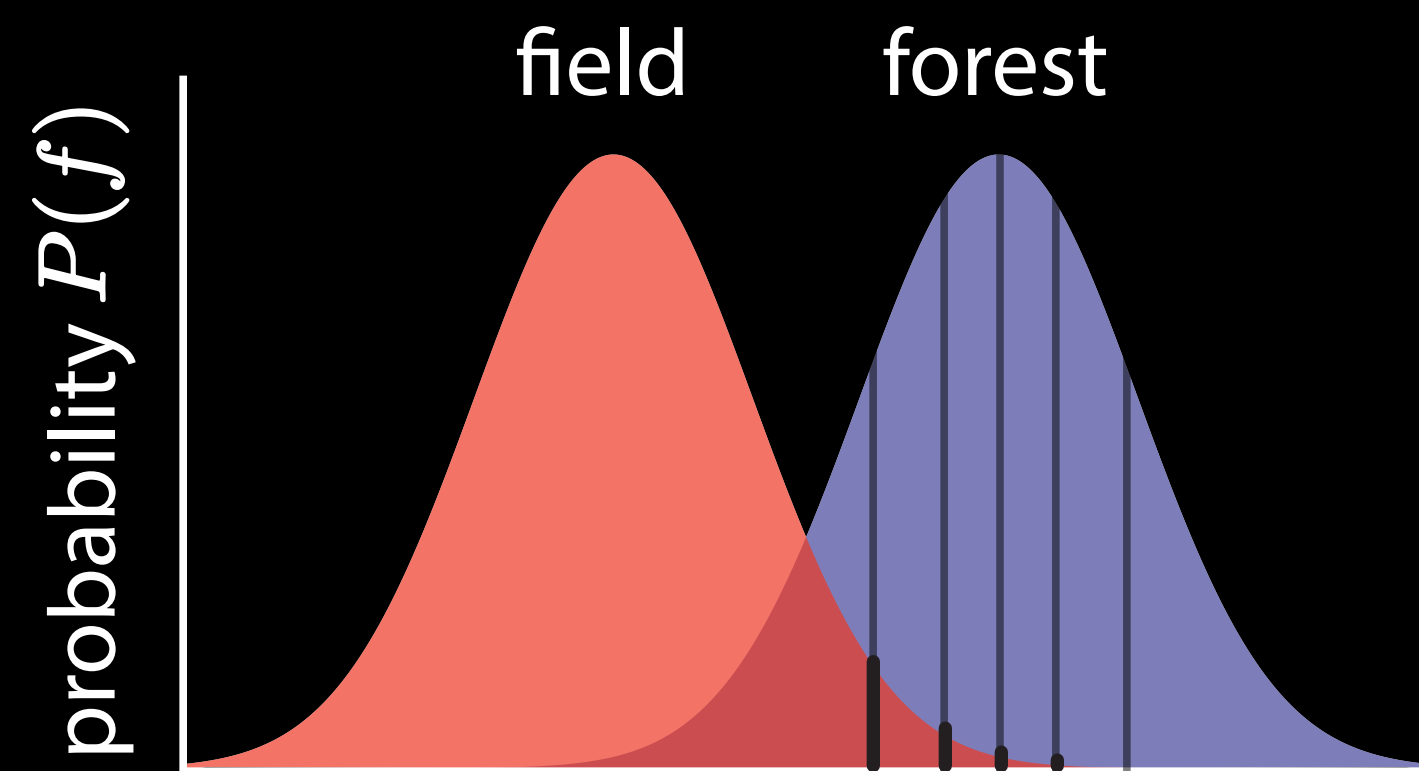
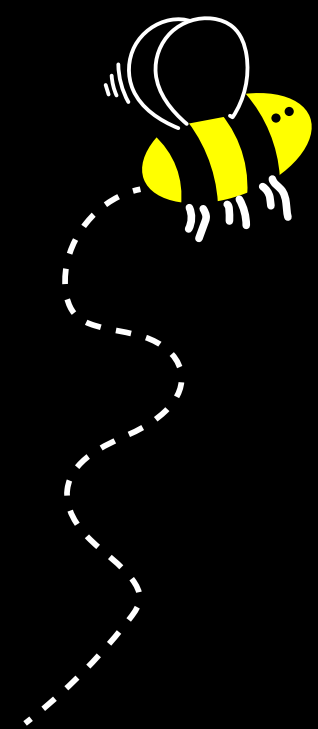


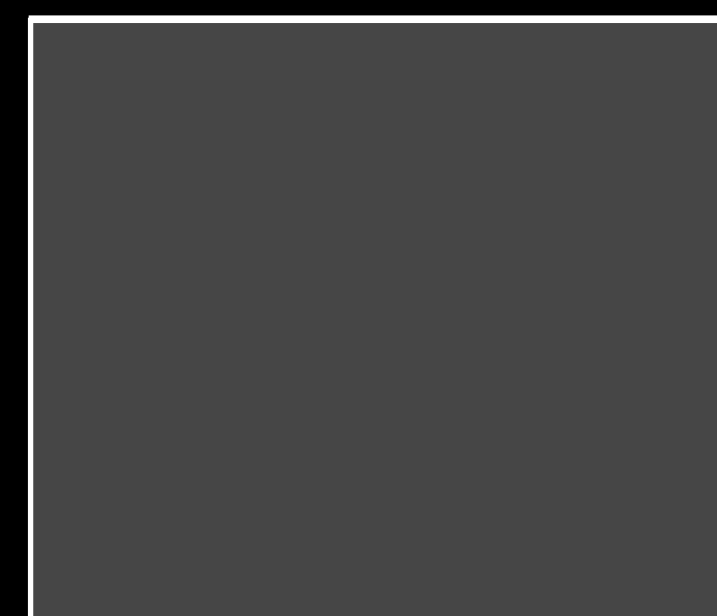
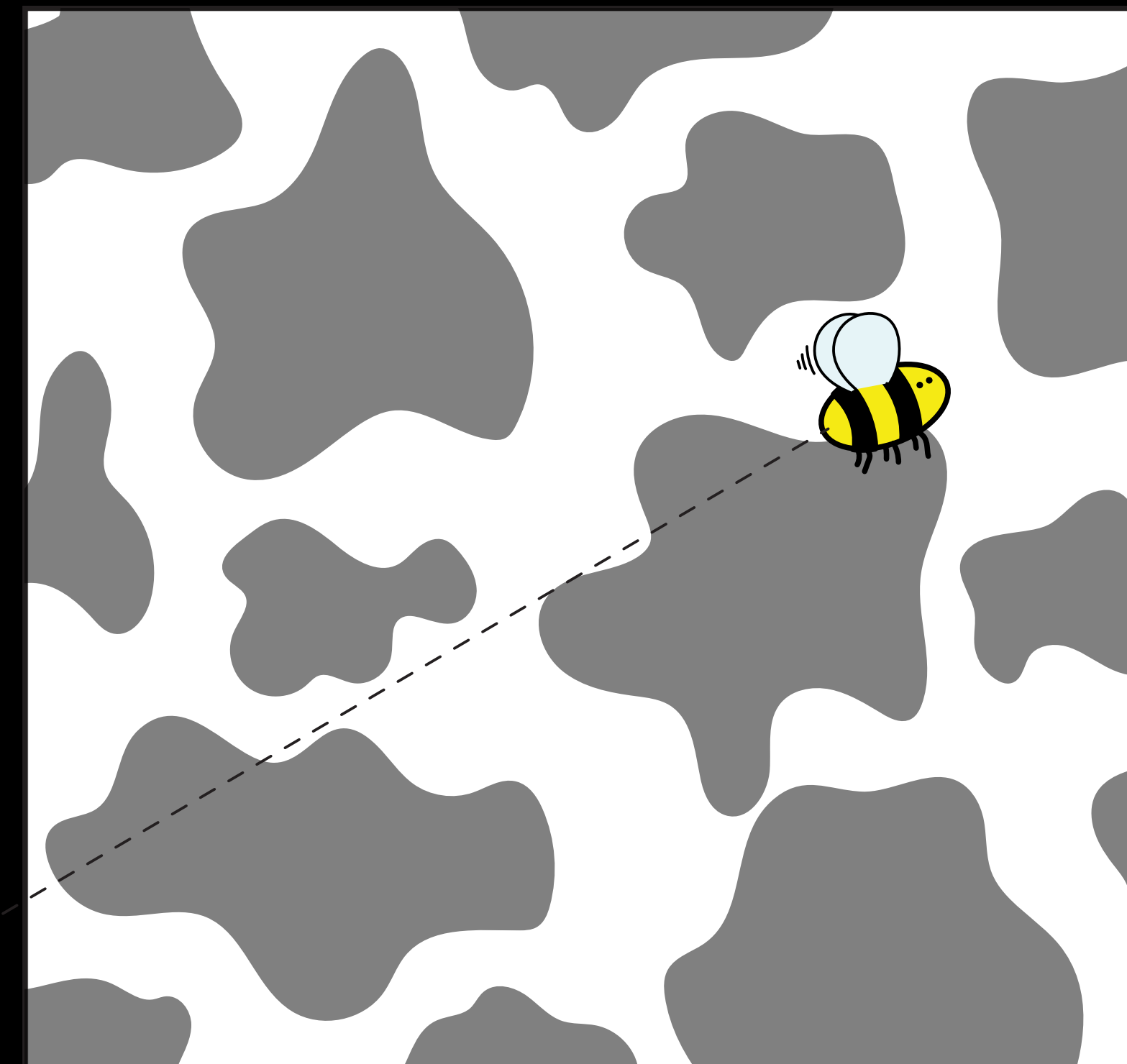
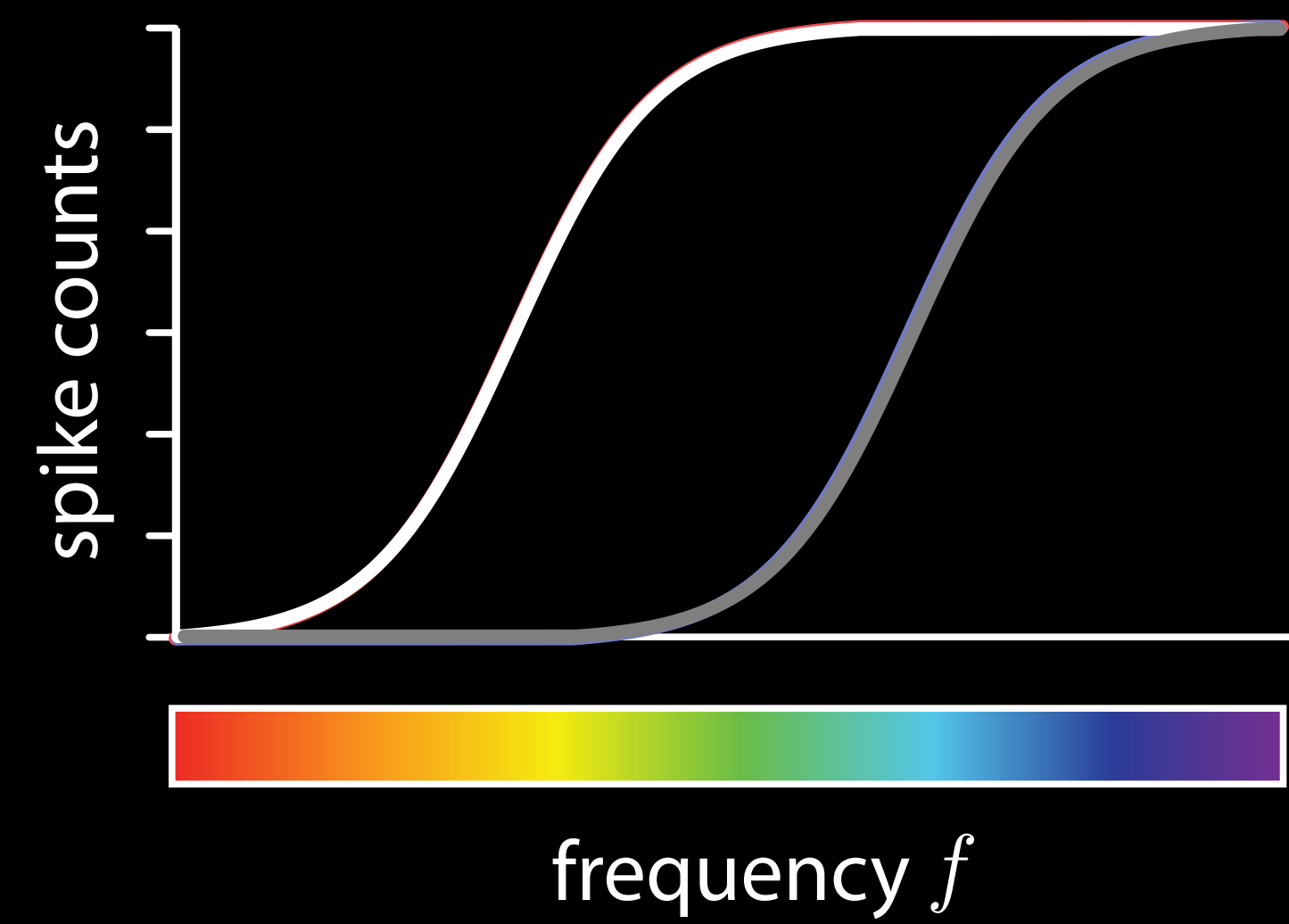
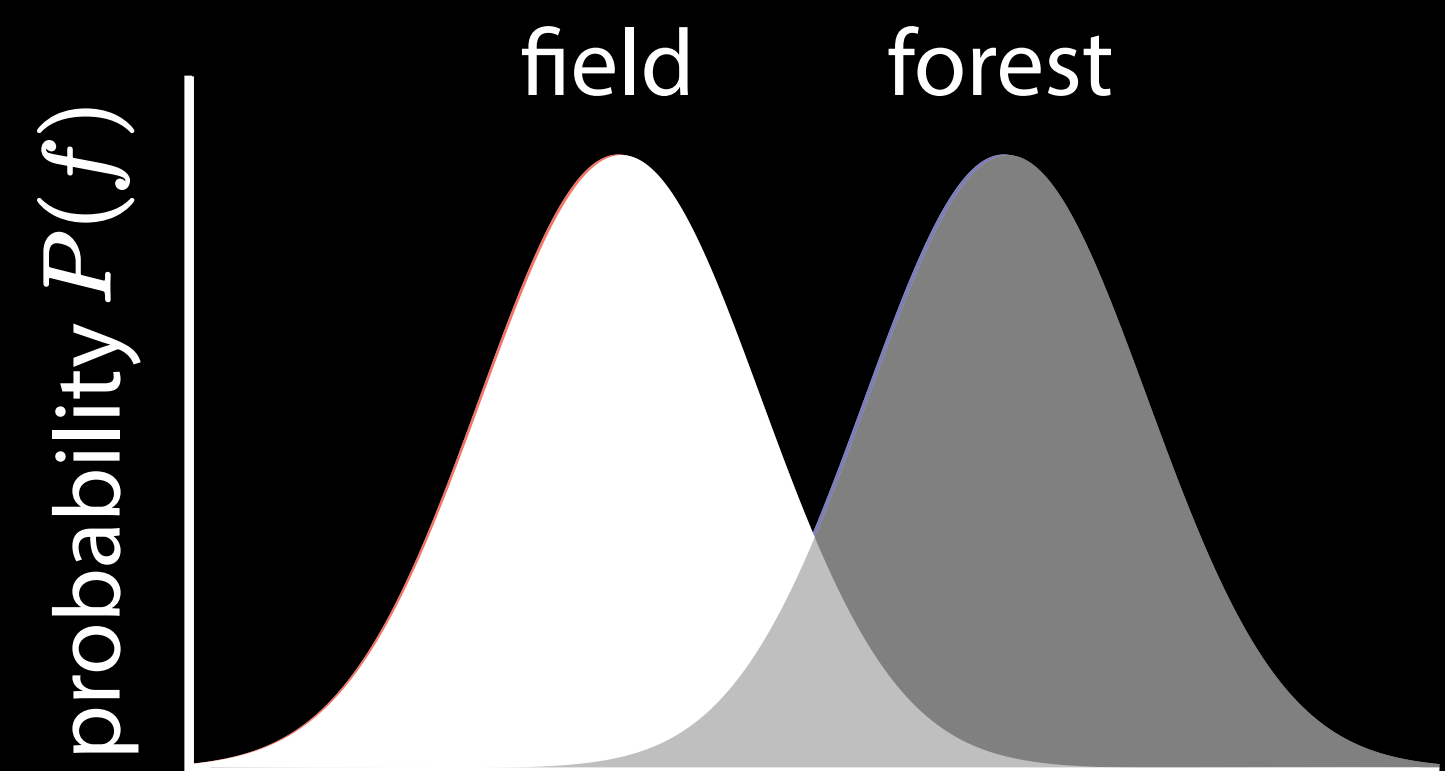
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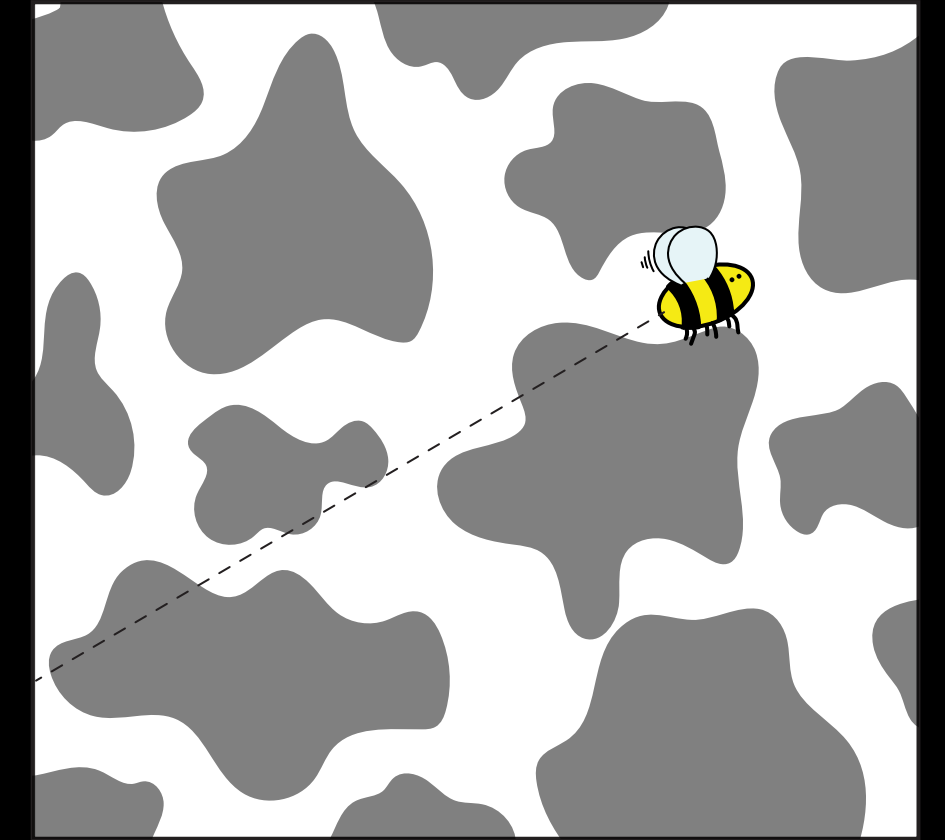
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field

context

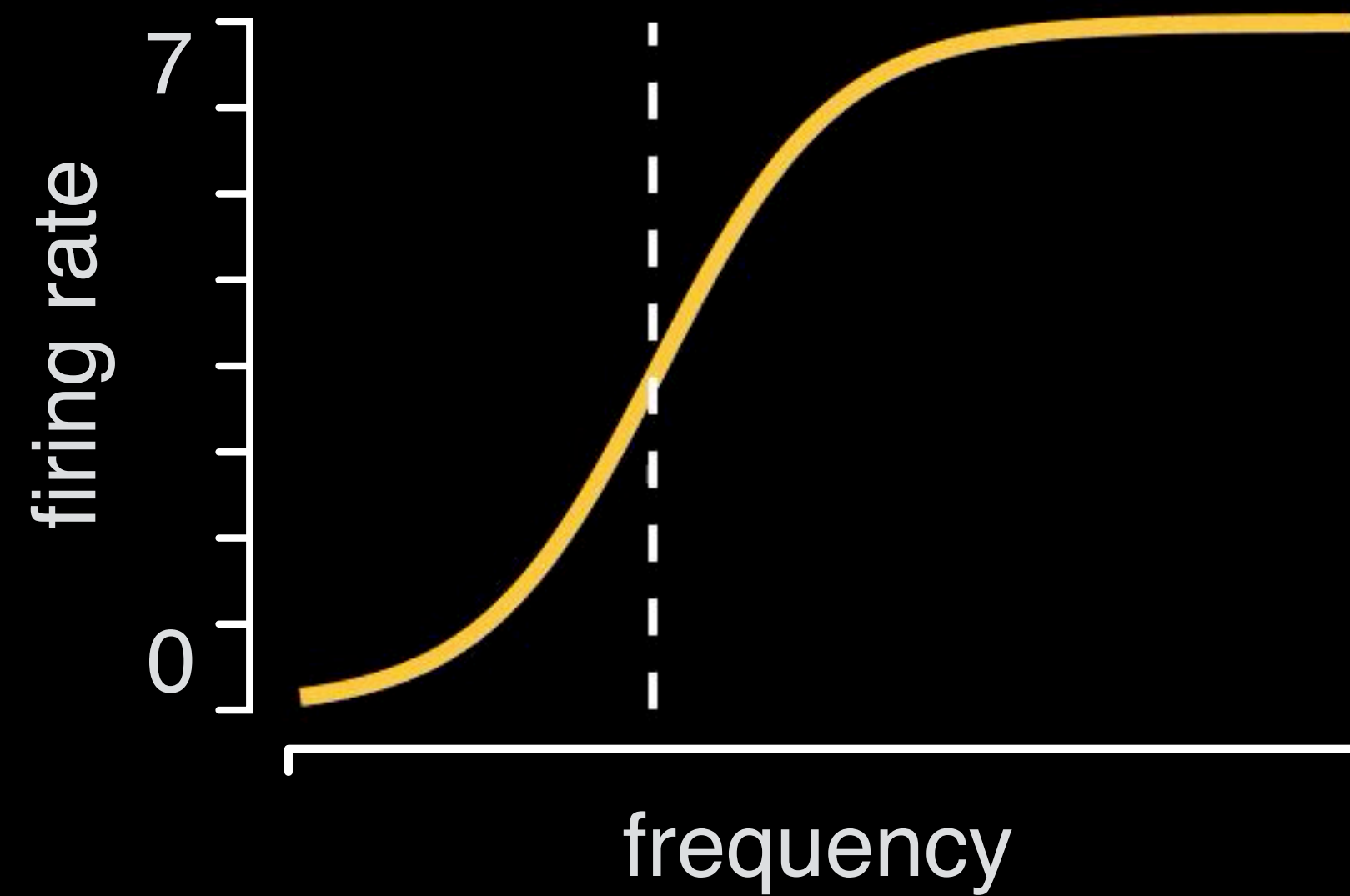
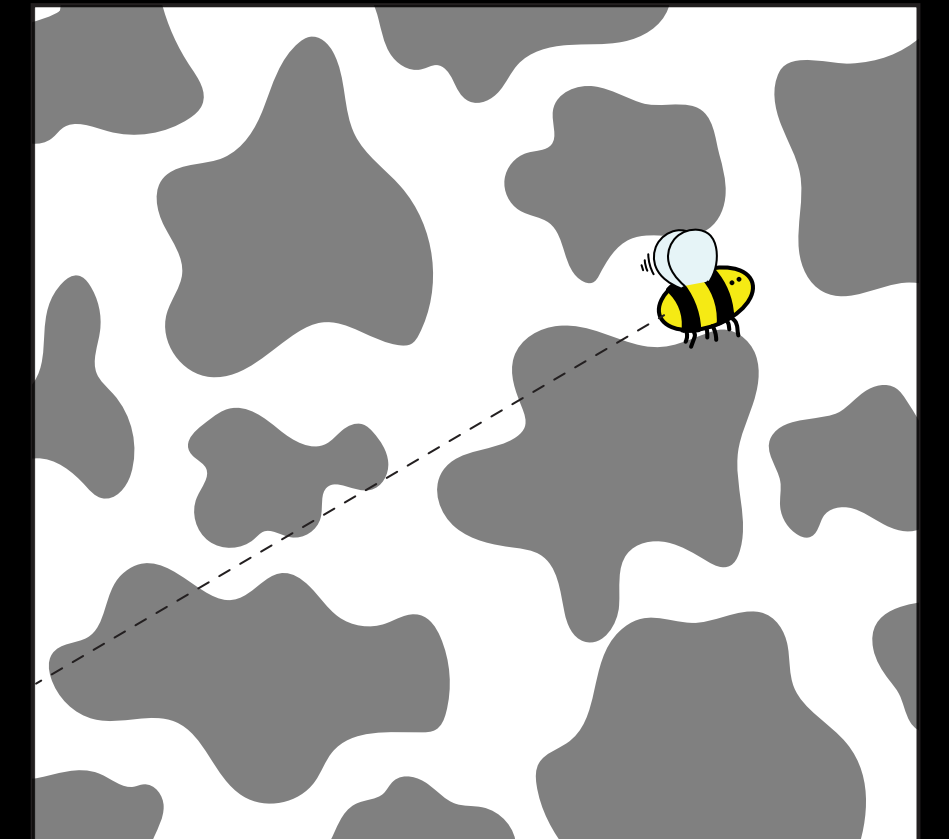
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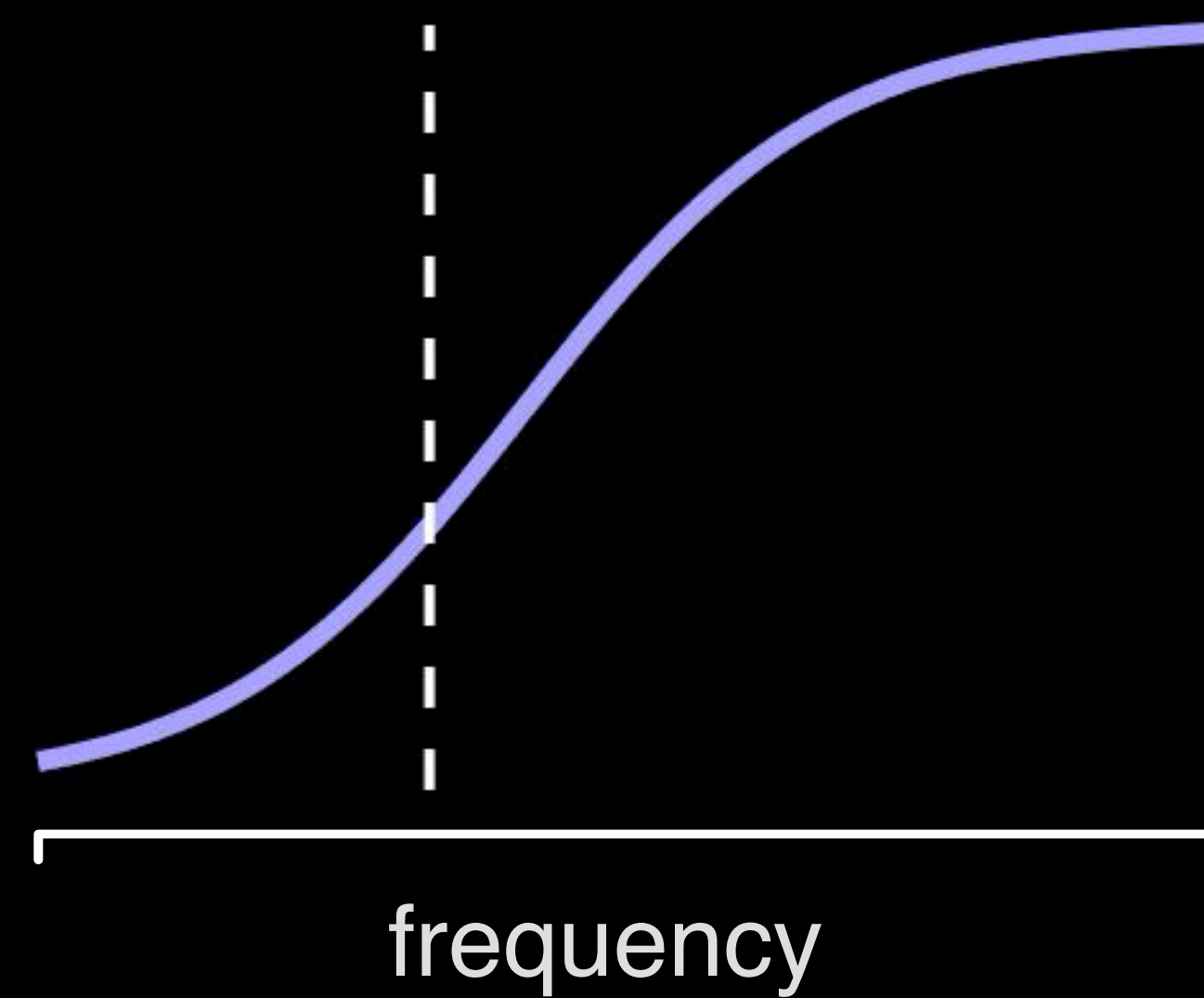
preserve information
about frequency

preserve information
about context

context

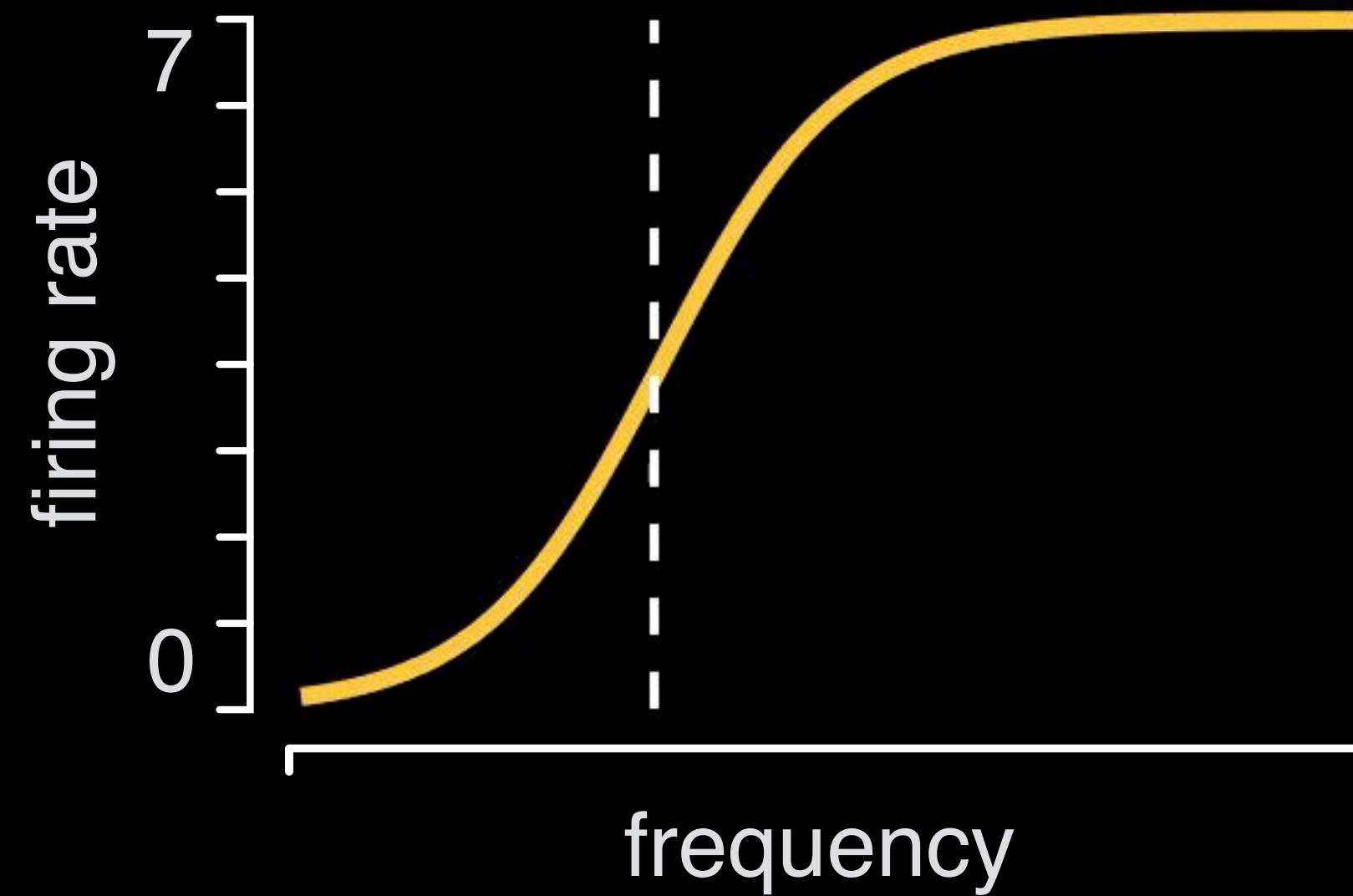
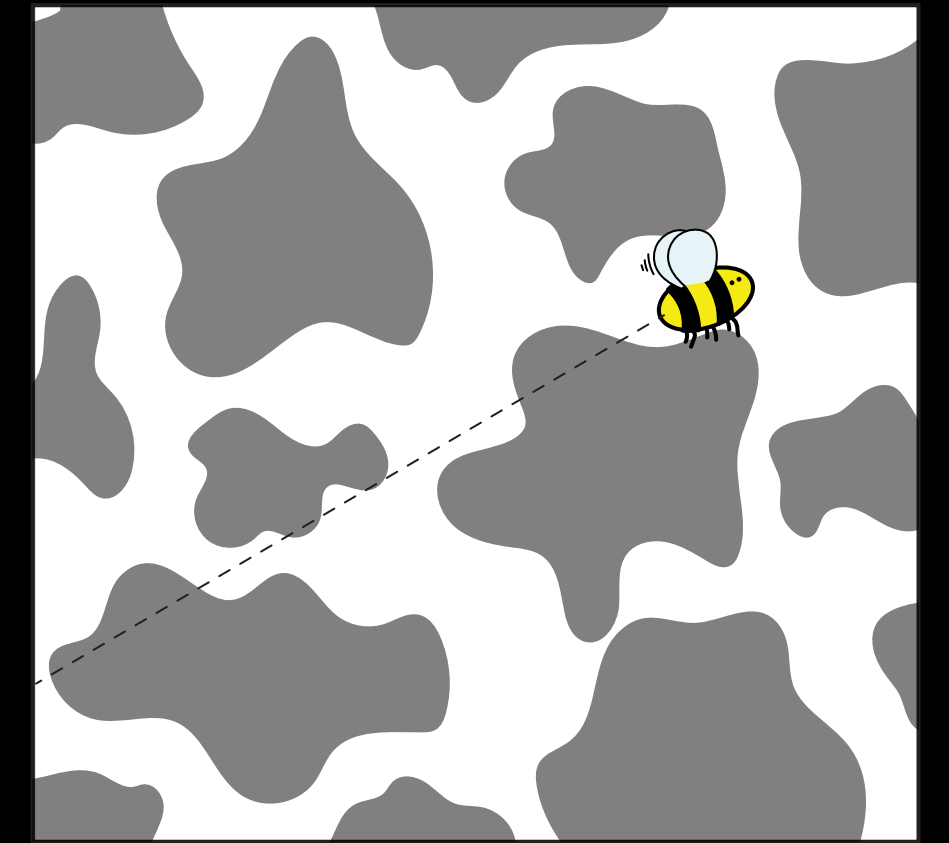


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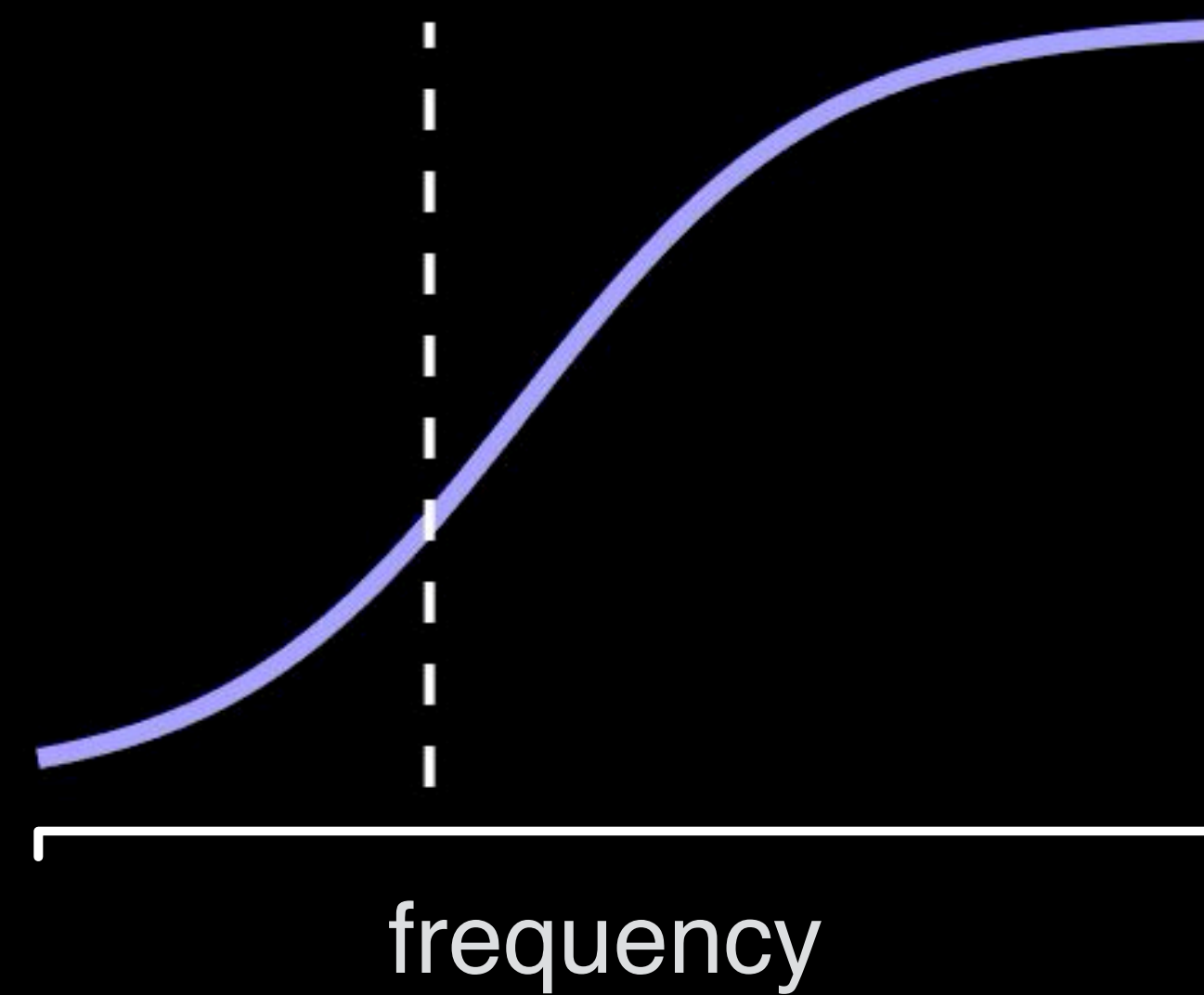


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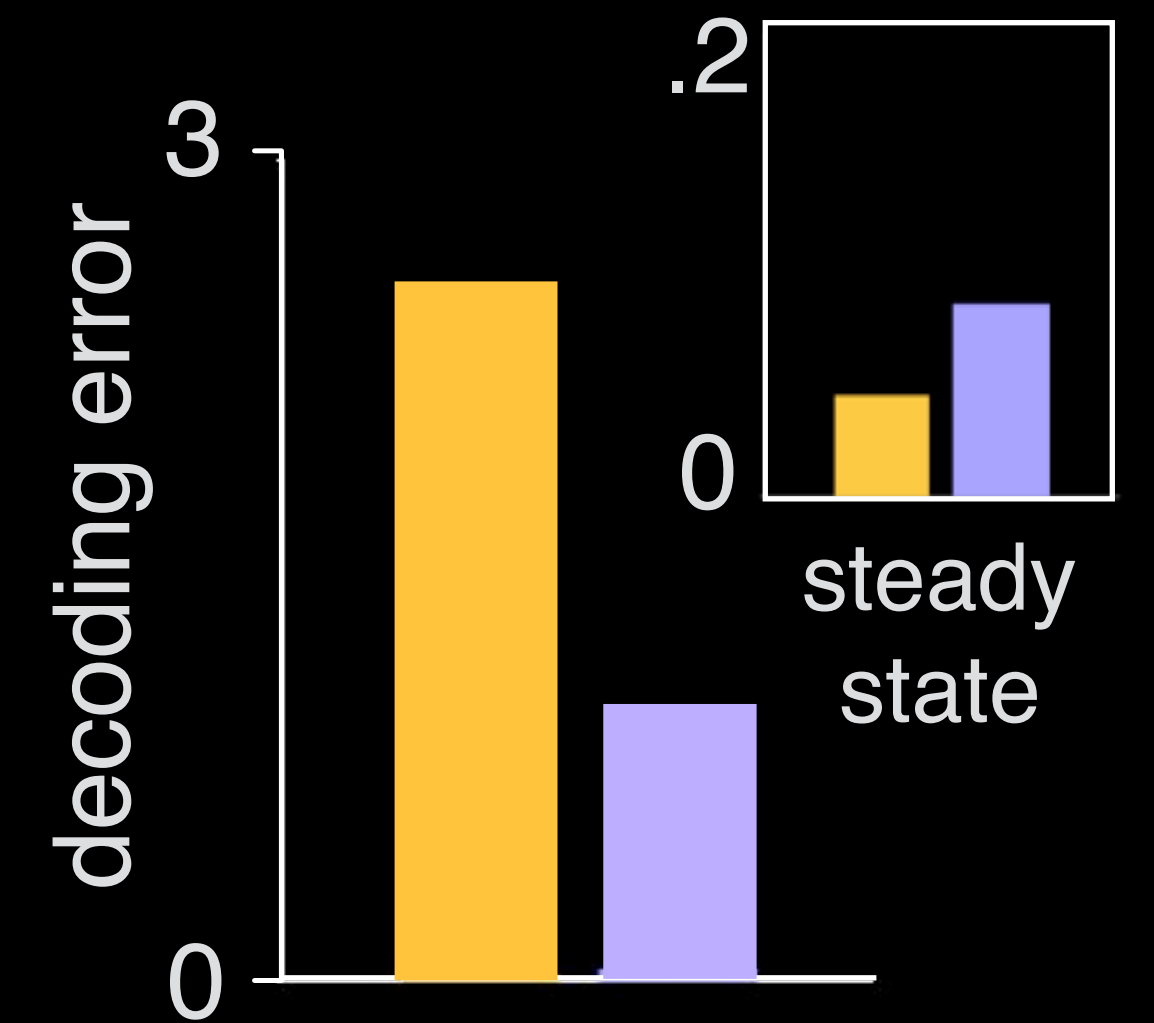
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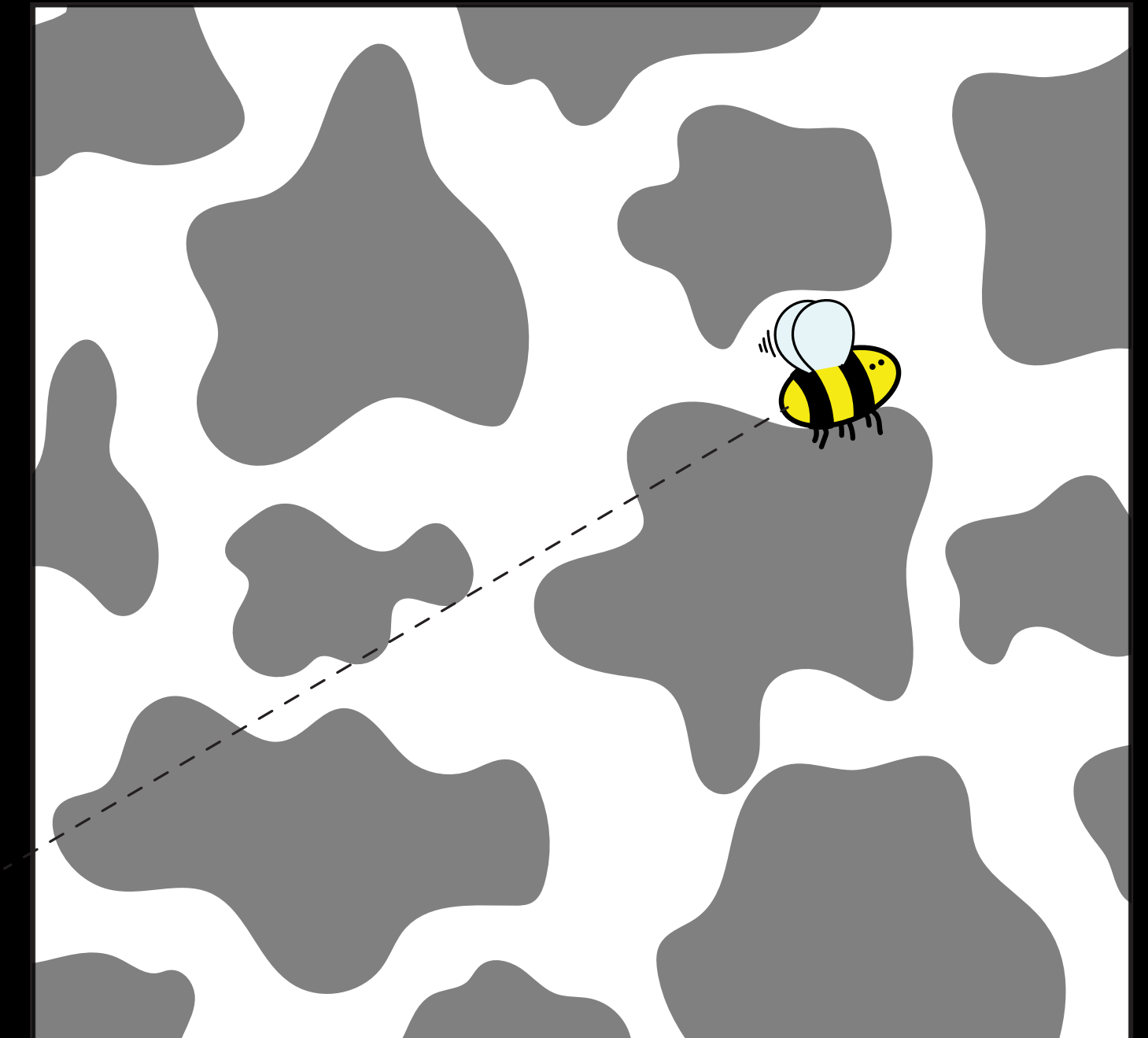
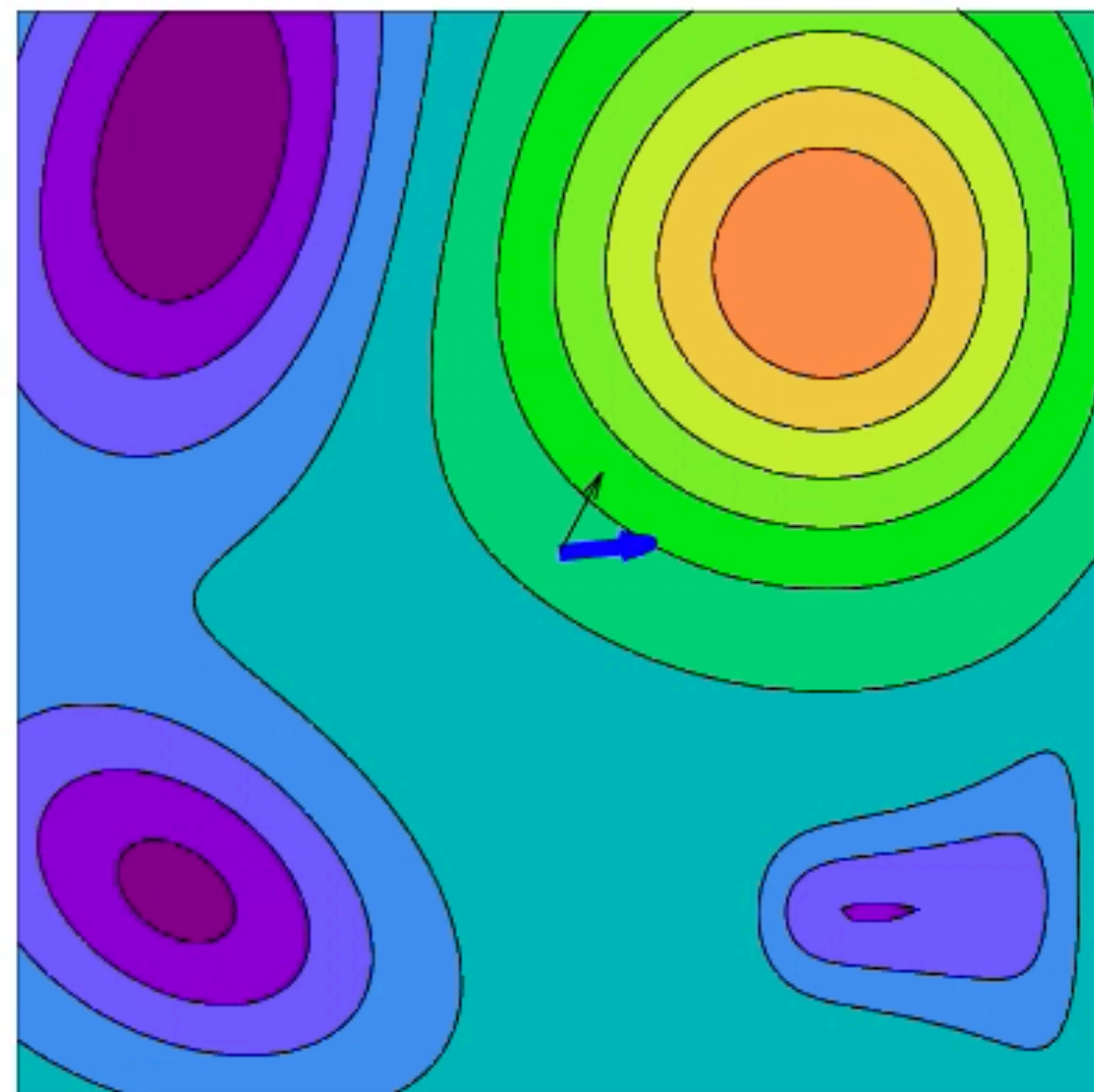
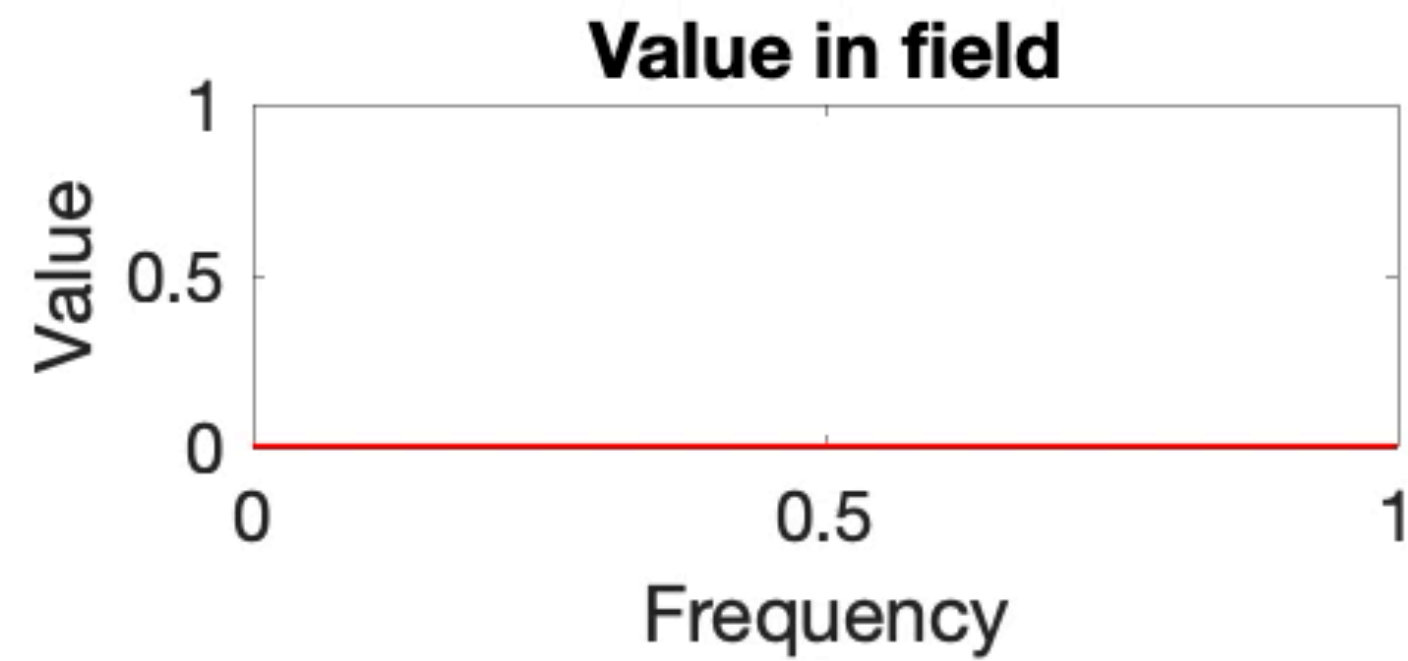
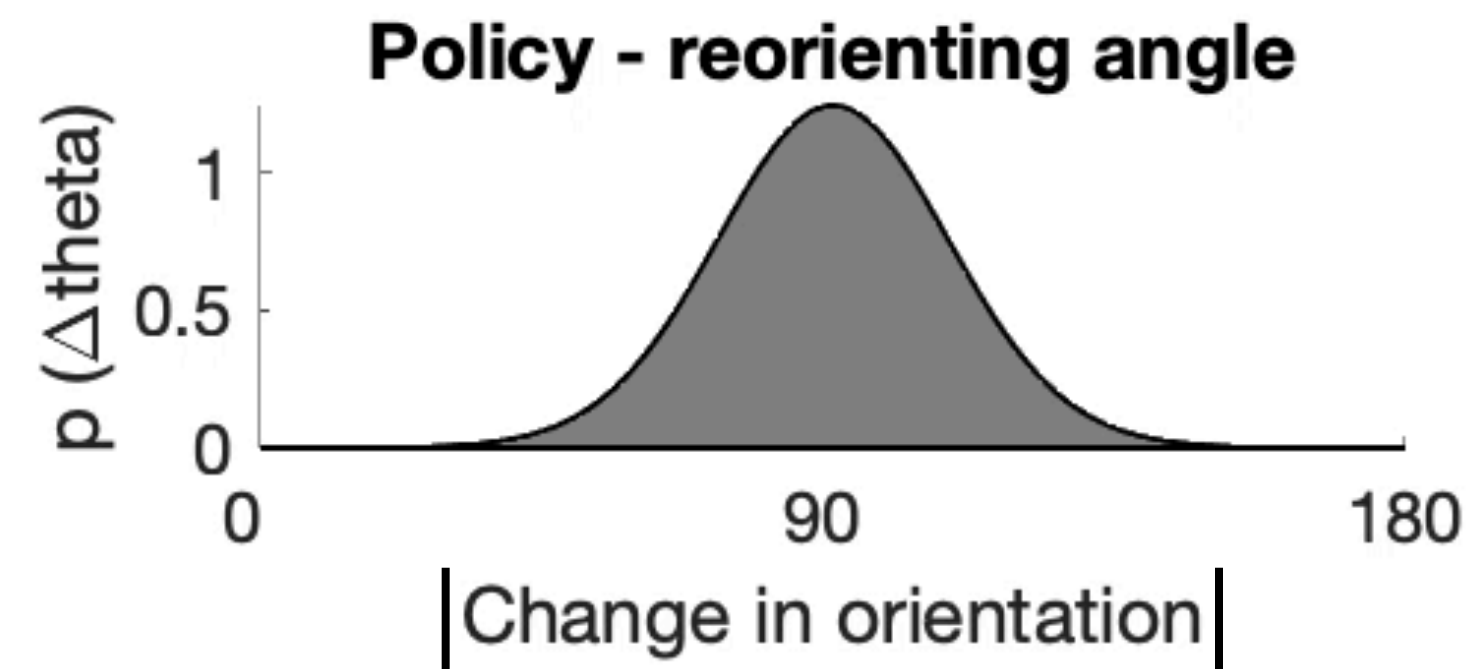
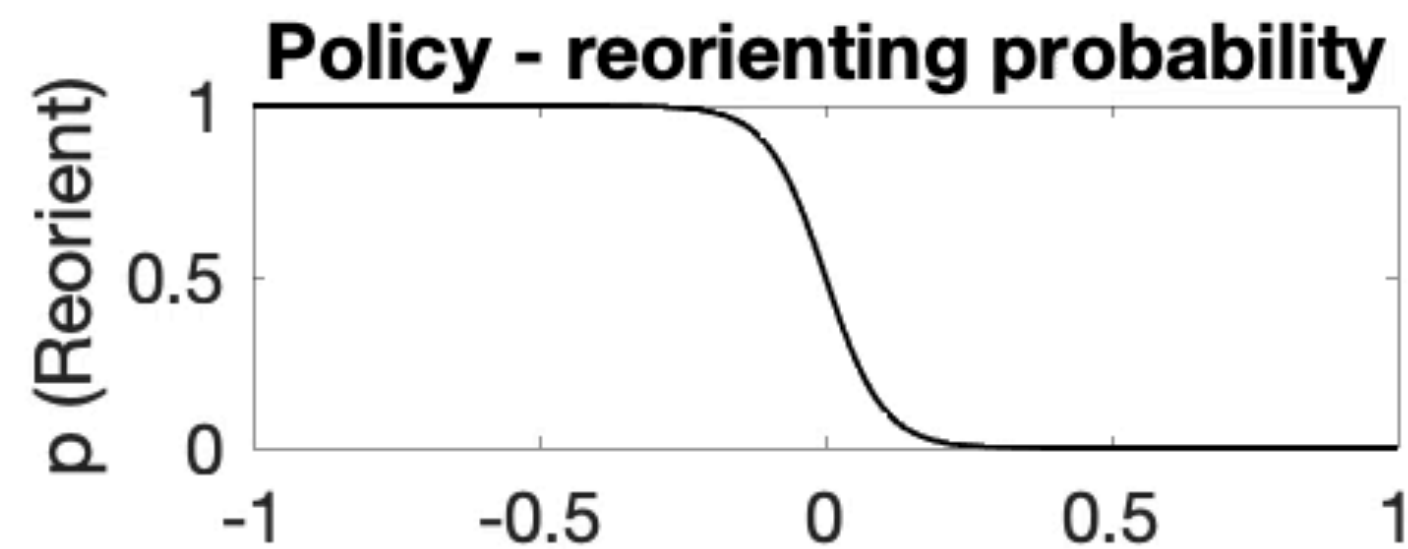


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about frequency

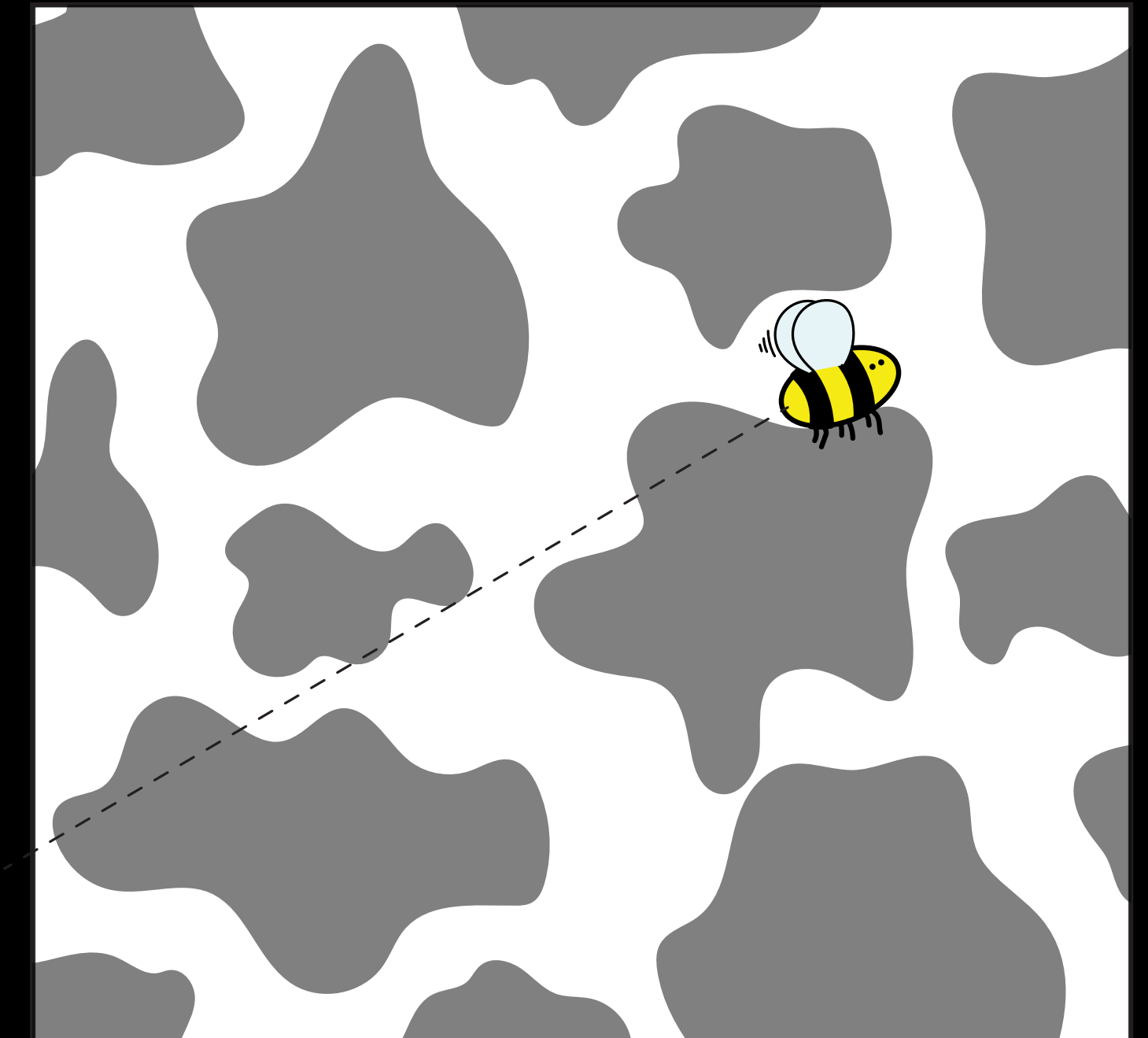
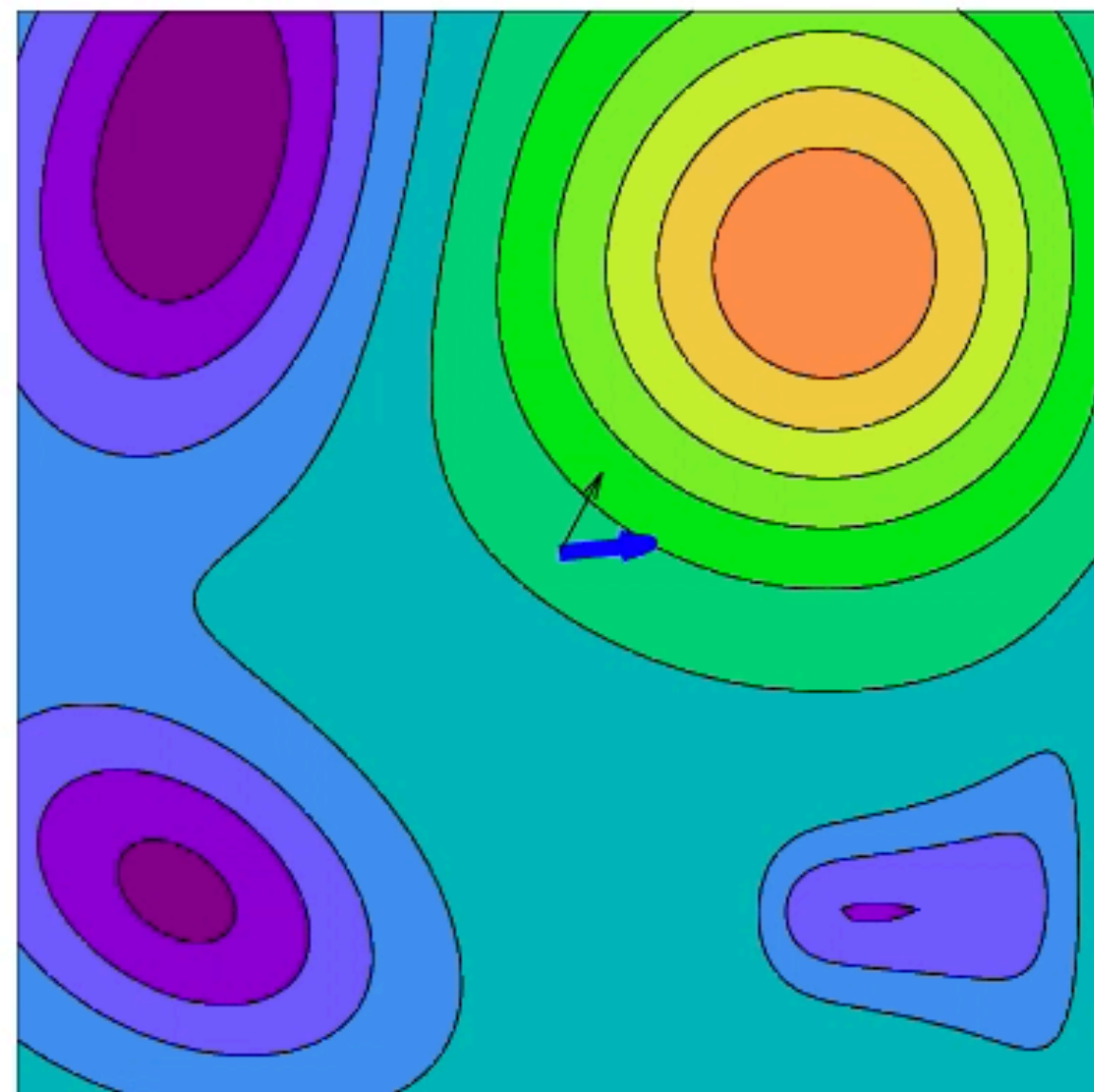
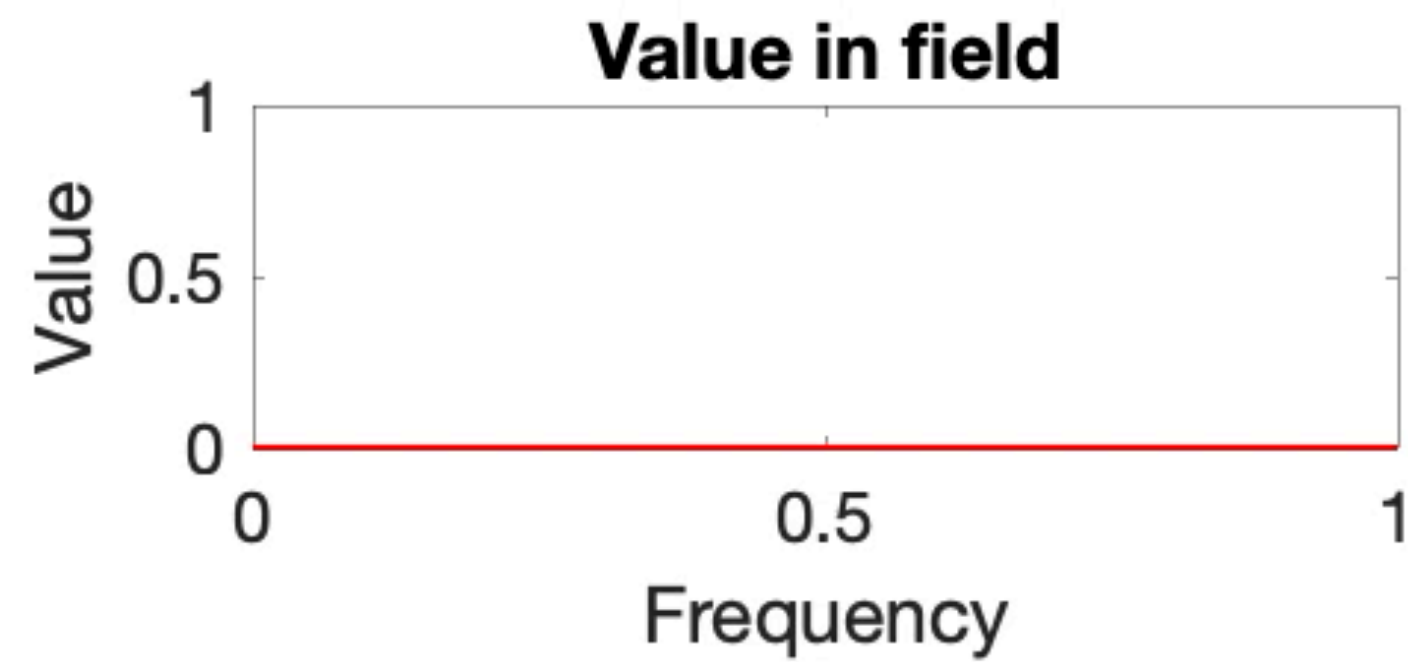
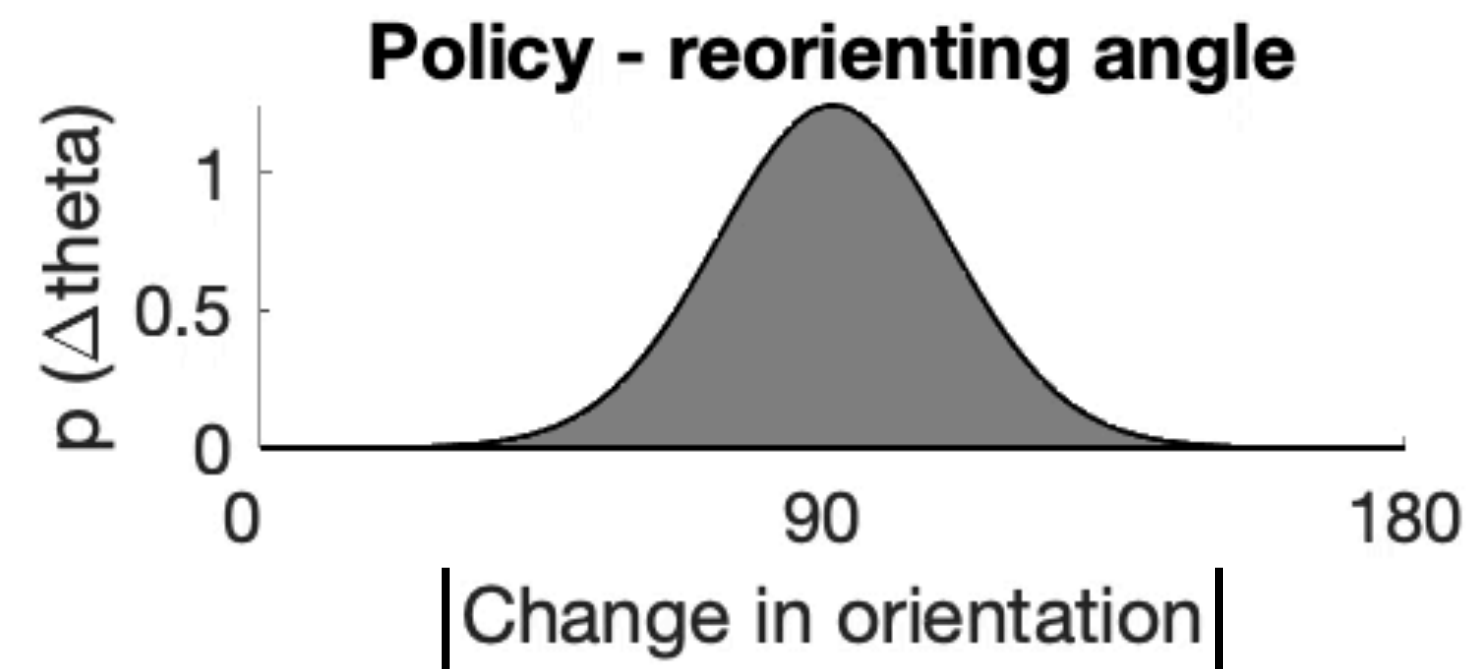
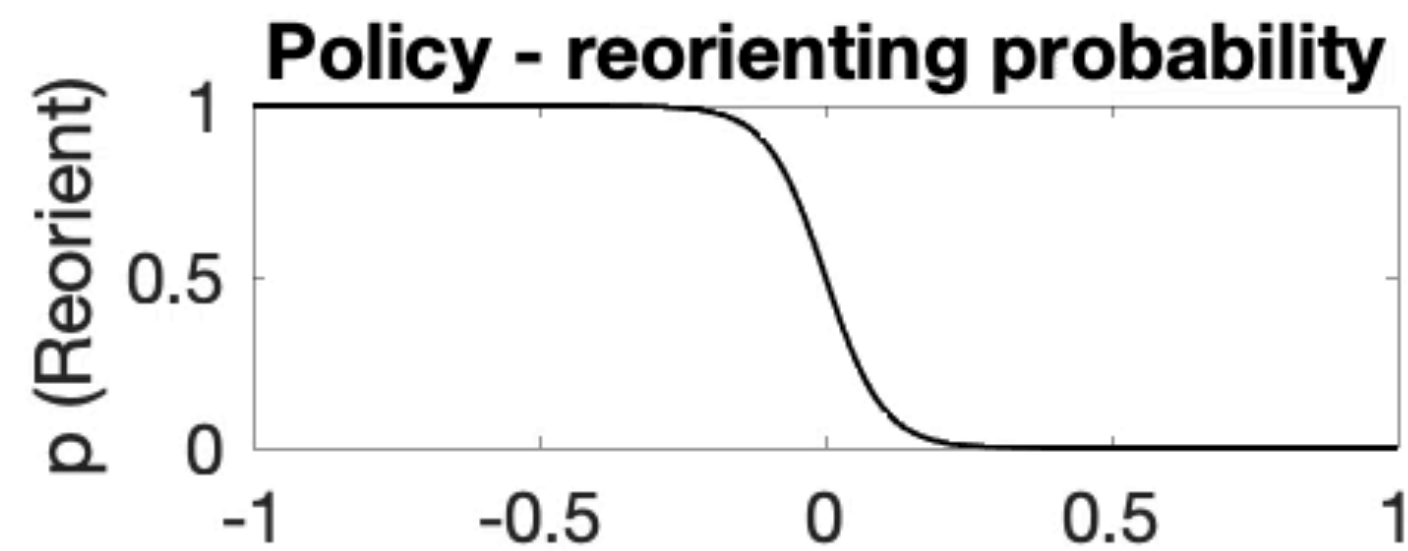


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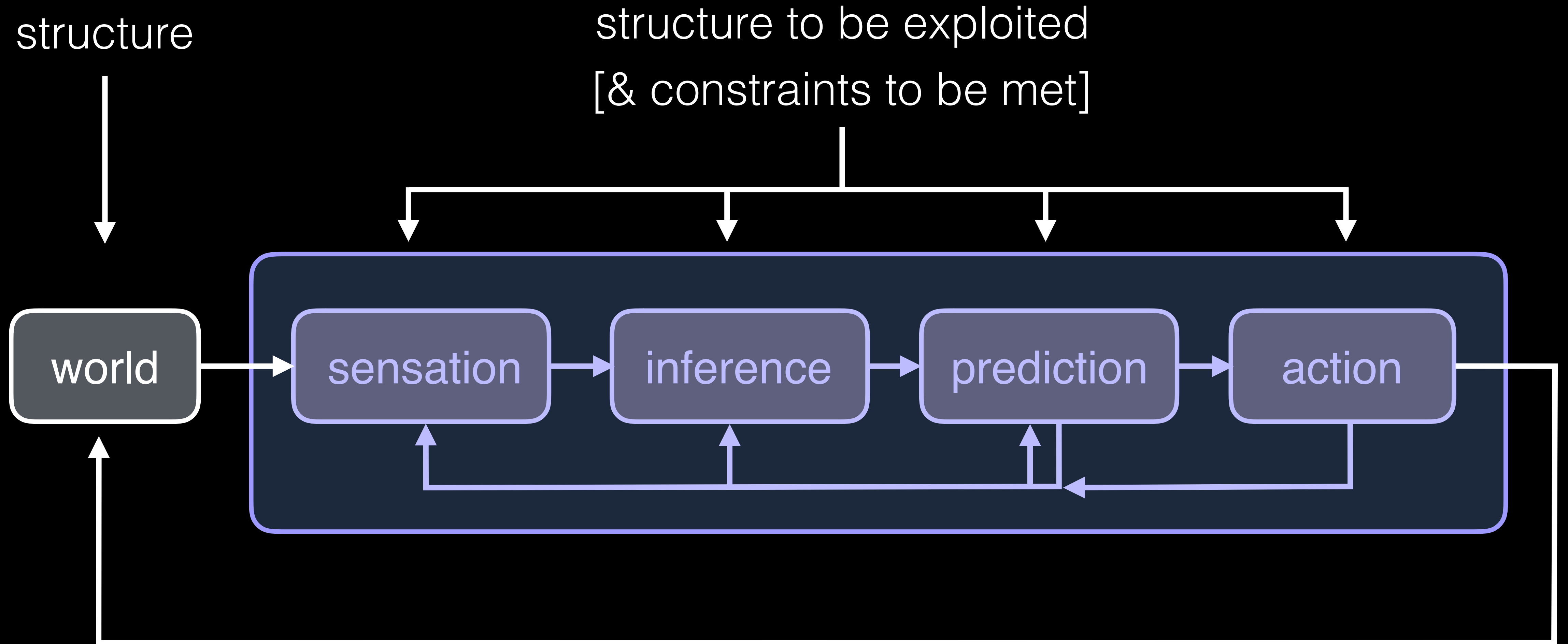




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